1. Introduction

From the complex algorithms used for image stabilization to the commonly used cruise control feature in cars, the application of electronic control is evident in many aspects of modern technology. Control theory is an essential toolbox for any engineer attempting to design a mechatronic system. By utilizing standardized methods for controller design such as negative feedback, PID control and lead-lag compensators, the engineer can ensure stability and optimal performance for the system. First, an open loop system is explored using a LRC circuit as the plant by modelling in MATLAB. Next, a negative feedback based system is created by routing the output into the input as an error signal. In the third section, circuits for proportional, integral and derivative controllers are designed separately and combined to create a tunable PID controller. Finally, implementation of lead-lag compensator circuits are explored. For each section various plots such as Bode, Nyquist and root locus are generated in MATLAB to offer insight into stability and the effects of varying parameters; LTSPice is used to test and confirm findings. The differences in results will be explained in detail in the discussions section.

2. Theory

This project will demonstrate real world applications of negative feedback control in electronic systems using LTSpice and MATLAB models. Concepts discussed in lectures will be thoroughly utilized to predict and confirm system behaviour.

2.1 LRC System Analysis

The first system is an Open Loop one created using a simple LRC circuit as shown in Figure 1 and 2.

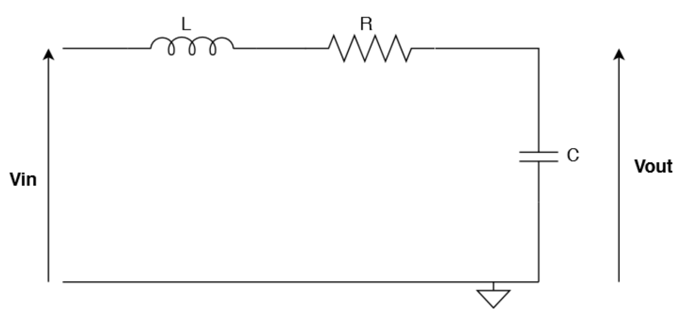


Figure 1: LRC Circuit

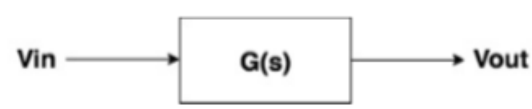


Figure 2: Open Loop Transfer Function

Using impedance, we may calculate the transfer function as:

$$G(s) = \frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_{in}} = \frac{Z_C}{Z_I + Z_R + Z_C} \tag{1}$$

By multiplying both the top and bottom by the term $\frac{s}{L}$ we obtain equation 2:

$$G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad (2)$$

The resulting transfer function is recognizable as a standard second order system. By putting in the given values for the inductor, capacitor and resistor, it is possible to simulate the physical system in MATLAB, and generate useful graphs as described below.

Step Response: Shows the system's response to a step input

Bode plot: Shows the frequency response of a system in terms of logarithmic magnitude and phase.

Root Locus: Shows system stability as a function of a gain parameter using the poles and zeros in the complex s-plane. This technique is used to develop stability criterion.

Nyquist: Shows the frequency response of a system with feedback. This technique is used to assess the stability of a system with a delay.

The resulting graphs will then be compared to ones generated in LTSpice by varying system parameters.

2.2 Negative Feedback

Next, we will develop a system that uses negative feedback to reach a desired outcome. This may be modeled by simply feeding the output back into the input of the system as shown in figure 3.

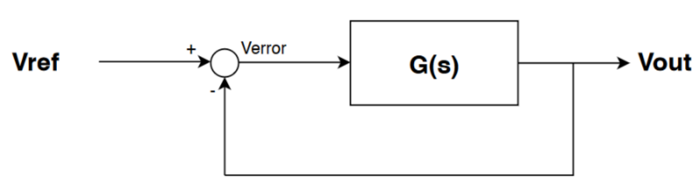


Figure 3: Negative Feedback System

In order to produce an electronic feedback output, an ideal op amp circuit may be employed as shown in figure 4. This configuration is typically called a differential amplifier.

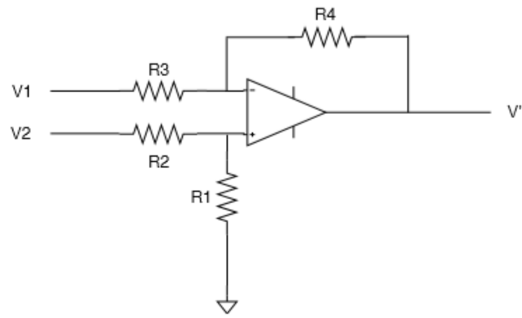


Figure 4: Negative Feedback in a Circuit

Using the principle of superposition the circuit may be solved by summing V' when each of the voltages are isolated and set to 0.

If V2 = 0, then the circuit becomes an inverter so

$$V'_1 = -V_1(\frac{R_4}{R_3}) \tag{3}$$

If V1 = 0, then the circuit becomes a non-inverting amplifier where Va is the voltage referred to ground at the non-inverting input of the operational amplifier

$$V'_2 = V_a\,(1 + \frac{R_4}{R_3}) \tag{4}$$

$$V_a = V_2\,(\frac{R_2}{R_1 + R_2}) \tag{5}$$

Finally, using the principle of superposition V' can be determined as the sum of V'1 and V'2

$$V' = -V_1(\frac{R_4}{R_3}) + V_2\,(\frac{R_2}{R_1 + R_2})(1 + \frac{R_4}{R_3}) \tag{6}$$

Taking all the resistors as having the same value, the final relationship is

$$V' = V_2 - V_1 \tag{7}$$

By relating it to the block diagram in figure 3 we can determine the analogous variables by

$$V_{out} = V_{ref} - V_{err} \tag{8}$$

Using this circuit, we can expect greater accuracy than the previous case due to the fact that the input is varied based on an error signal. This prediction will be confirmed in the results using MATLAB simulations.

2.3 Controller Design

By combining the first two circuits, it is possible to create a negative feedback system with a controller as shown in Figure 5

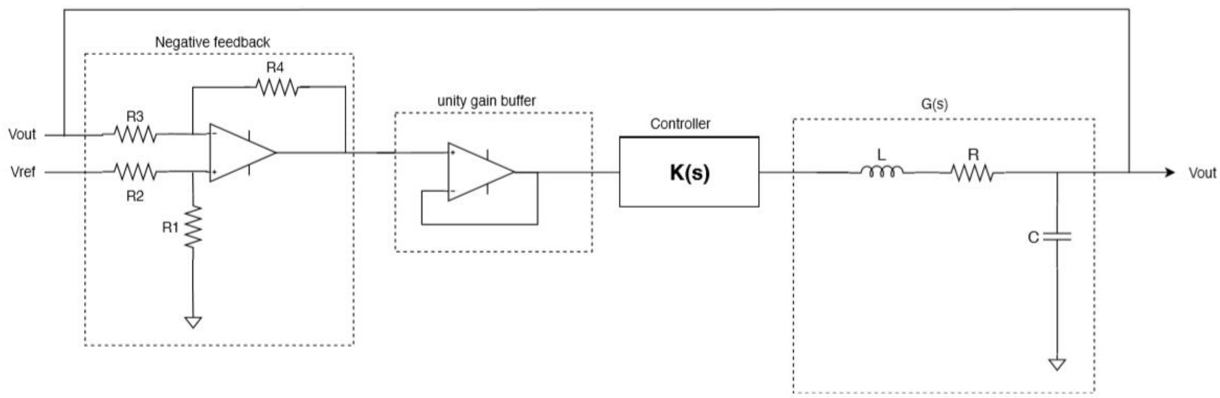


Figure 5: Negative Feedback System with Controller - Circuit Implementation

The following subsections will explore two different designs for K(s): PID controller and Lead/Lag compensator. MATLAB will be used to assess the performance of the overall system in terms of stability, response time, oscillatory behaviour, and steady state offset. The final designs will be tested out in LTSPice

2.3.1 PID Control

A PID controller is made of three components; a proportional, integral and derivative controller. When combining these three we are able to control all attributes of a response signal in terms of overshoot, stability, steady-state, and response time. Initially, each circuit will be explored separately before being combined as shown in figure 6.

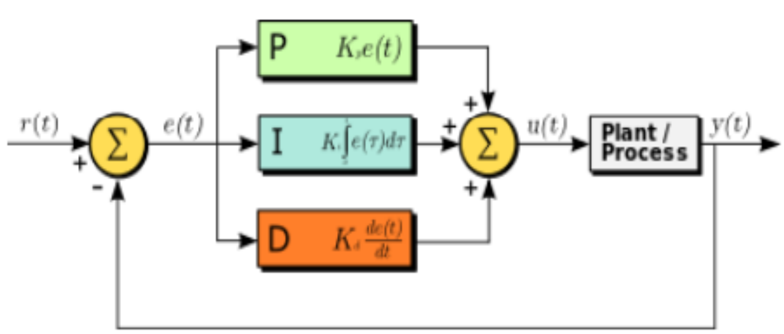
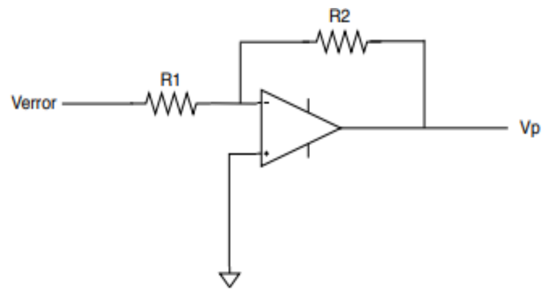


Figure 6: PID Controller

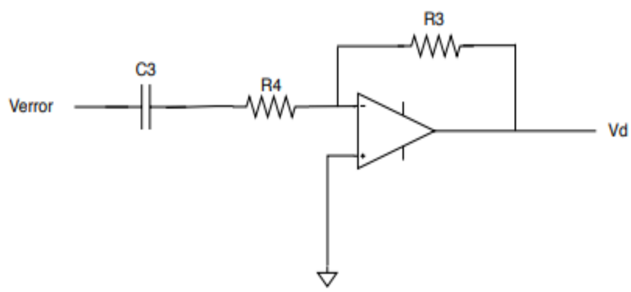
First, a proportional controller may be designed using an op-amp circuit as shown in figure 7.



$$K_p = -\frac{R_2}{R_1} \tag{9}$$

Figure 7: Proportional Op-Amp Circuit

Next, a derivative controller is created by adding a capacitor to the proportional op amp circuit as shown in figure 8.



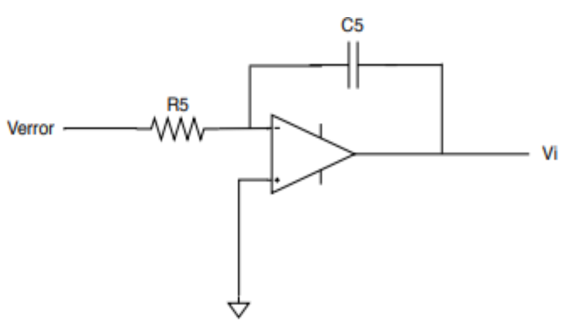
$$K_d = -\frac{R_3 C_3 s}{R_4} \tag{10}$$

Figure 8: Differential Op-Amp Circuit

If we take R_4 to be much smaller than R_3 then the equation can be simplified to:

$$K_d = -R_3 C_3 s \tag{11}$$

By moving the capacitor into the feedback path and replacing the resistor, it is possible to create an integral controller as shown in figure 9.



$$K_i = -\frac{1}{R_5 C_5 s} \tag{12}$$

Figure 9: Integral Op-Amp Circuit

Finally, all 3 controllers are combined using the summing circuit illustrated in figure 10. Equation 15 shows the final relationship between

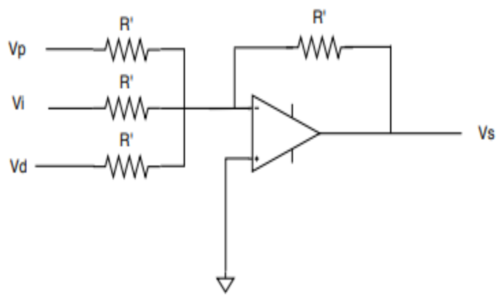


Figure 10: PID Gain Circuit

$$V_s = -(R'I')\,V_{err} = -R'\,(I_p + I_i + I_d)\,V_{err} \tag{13}$$

$$V_s = -R'\,(\frac{V_p}{R'} + \frac{V_i}{R'} + \frac{V_d}{R'})\,V_{err} \tag{14}$$

$$V_s = -(V_p + V_i + V_d)\,V_{err} \tag{15}$$

Using MATLAB, the PID controller is tuned to obtain the most favorable response by varying values of Kp, Kd and Ki.

2.3.2 Lead-Lag Compensation

Another way to tune the performance of a system is by improving the gain and phase margins using a lead and/or lag compensator circuit.

2.3.2.1 Lag Compensator Design

A lag compensator adds a negative phase to the system over a specified frequency range. The main effect that is brought about by a lag compensator is that it adds gain to the system at low frequencies. The magnitude of the gain is equal to 'α'.The effect brought about by this gain is that it lowers the steady-state error of the closed-loop system by a factor of 'α'.

The negative effect of the lag compensator is the negative phase that is between the two corner frequencies - the specified frequency range that is affected by the addition of the lag compensation. The negative phase added can increase up to -90 degrees. Therefore, care must be taken to ensure that the phase margin of the open-loop system with the lag compensator is still satisfactory.

A lag compensator may be implemented as shown in Figure 11.

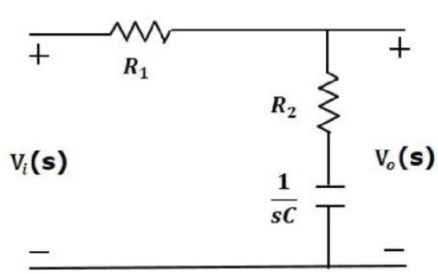


Figure 11: Lag Compensator

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\left(\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}\right) \tag{16}$$

Where $\tau = R_2C$ and $\alpha = \frac{R1+R2}{R2}$

Simplifying the equation we can obtain:

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1 + R_2}\left(\frac{Cs + \frac{1}{R_2}}{Cs + \frac{1}{R_1 + R_2}}\right) \tag{17}$$

By setting $R_1 = 1k\Omega$, $R_2 = 1k\Omega$, and $C_2 = 1\mu F$, the transfer function may be calculated for the first MATLAB simulation case as

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{2}\left(\frac{s + 1000}{s + 500}\right) \tag{18}$$

By setting $R_1 = 2k\Omega$, $R_2 = 1k\Omega$, and $C_2 = 1\mu F$, the transfer function may be calculated as for the first MATLAB simulation case

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{3}\left(\frac{s + 1000}{s + 333.33}\right) \tag{19}$$

2.3.2.2 Lead Compensator Design

A lead compensator adds a positive phase to the system over a specified frequency range. The main effect that is brought about by a lead compensator is that it adds gain to the system at high frequencies. The magnitude of this gain is equal to 'β'. The effect brought about by this gain is that it increases the crossover frequency, which help decrease rise time and settling time of the system, allowing a faster transient response.

The negative effect of a lead compensator is the added gain to high frequencies which results in the undesirable amplification of high frequency noise. Also, the addition of the positive phase helps improve the phase margin of the system, thus improving the stability of the system.

The circuit implementation of a lead compensator is explored as illustrated in Figure 12.

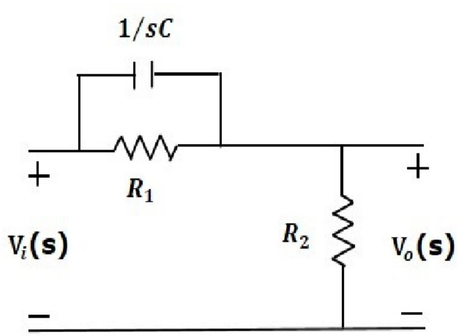


Figure 12: Lead Compensator

$$\frac{V_o(s)}{V_i(s)} = \beta\left(\frac{s\tau + 1}{\beta s\tau + 1}\right) \tag{20}$$

Where $\tau = R_1C$ and $\beta = \frac{R2}{R1+R2}$

As with the lag circuit, by simplifying the equation we can obtain:

$$\frac{V_o(s)}{V_i(s)} = \frac{(R_1 R_2 C)s \ + \ R_2}{(R_1 R_2 C)s \ + \ R_1 \ + \ R_2} \tag{2}$$

By setting R1 = 1kΩ, R2 = 1kΩ, and C2 = 1μF, the transfer function may be calculated for the MATLAB simulation as

$$\frac{V_o(s)}{V_i(s)} = \frac{s \ + \ 1000}{s \ + \ 2000} \tag{22}$$

Finally, by combining both circuits with a proportional controller a lead lag compensator is created as shown in Figure 13.

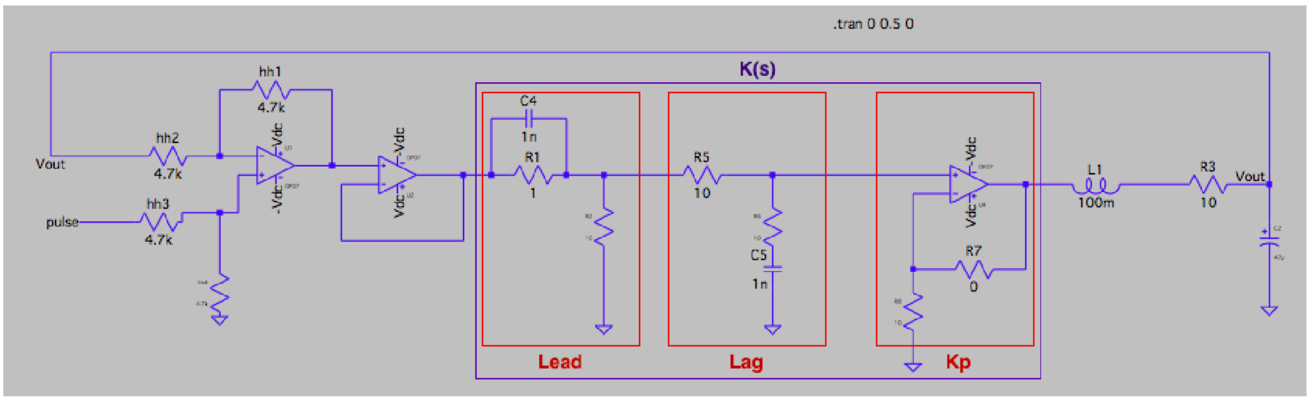


Figure 13: Lead Lag Compensator LTSpice Model

And the equation for the final signal may be obtained by applying block diagram rules as:

$$K(s) = \beta \left(\frac{s\tau + 1}{\beta s\tau + 1} \right) \frac{1}{\alpha} \left(\frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}} \right) K_p \tag{23}$$

3. Results

3.1 LRC System Analysis

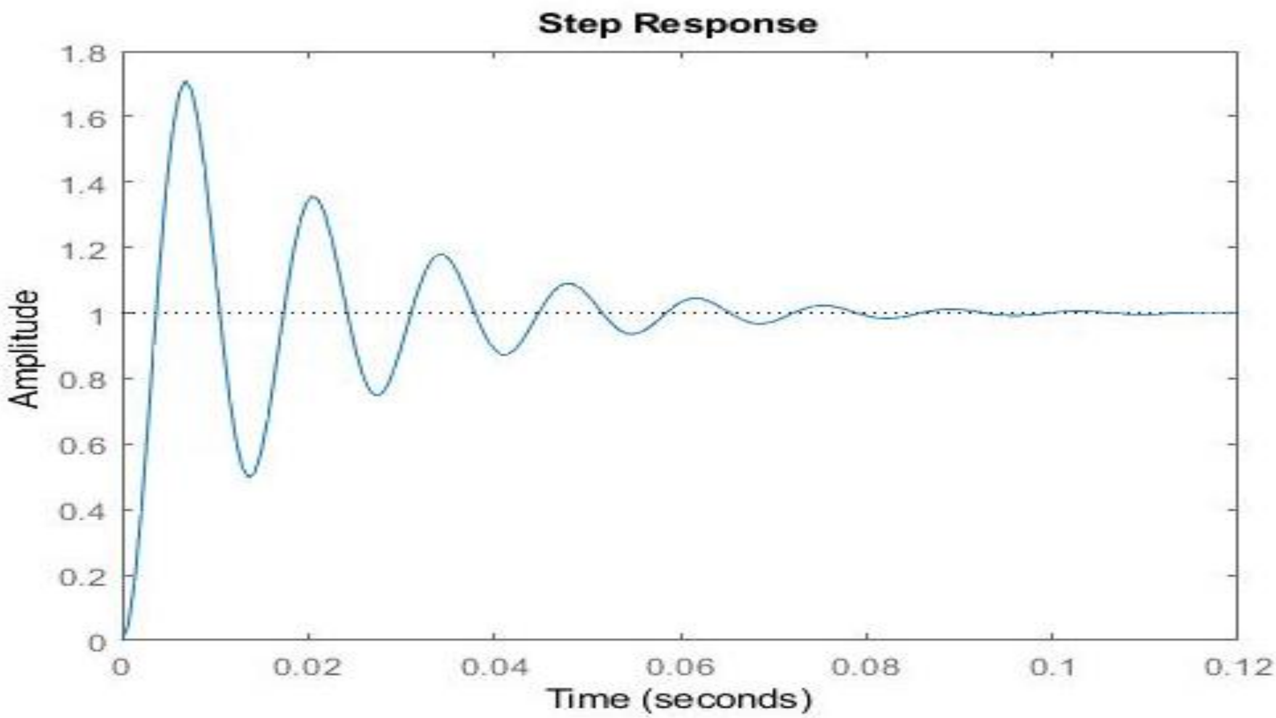


Figure 14: LRC step response

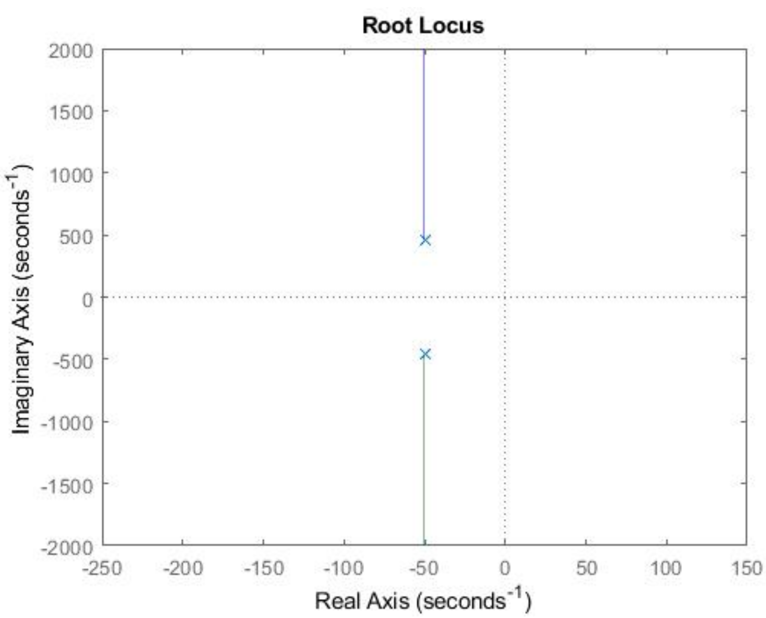


Figure 15: LRC Root Locus

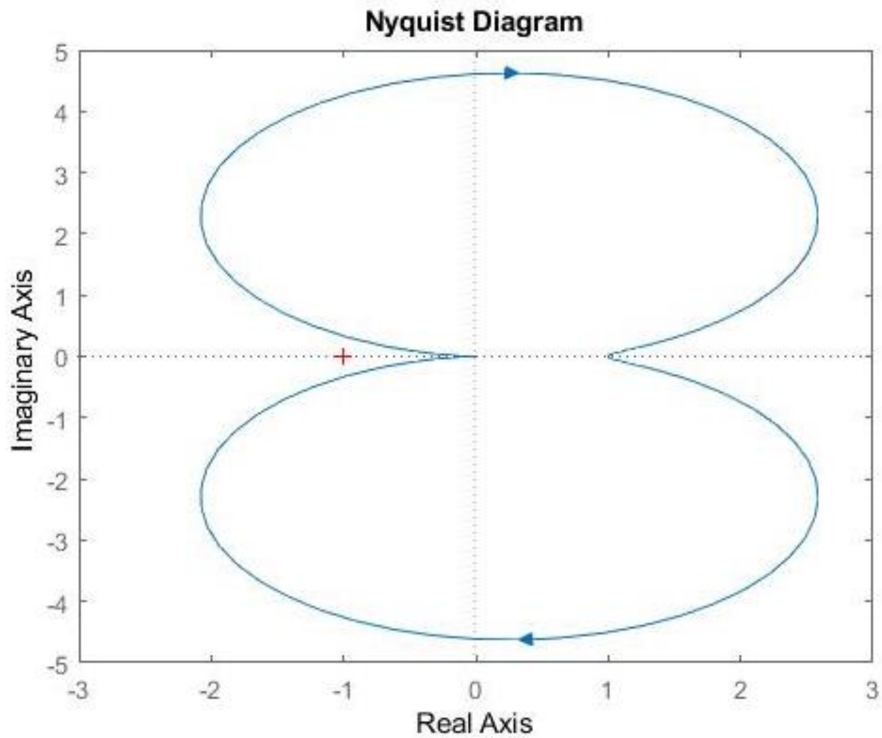


Figure 16: LRC Nyquist Plot

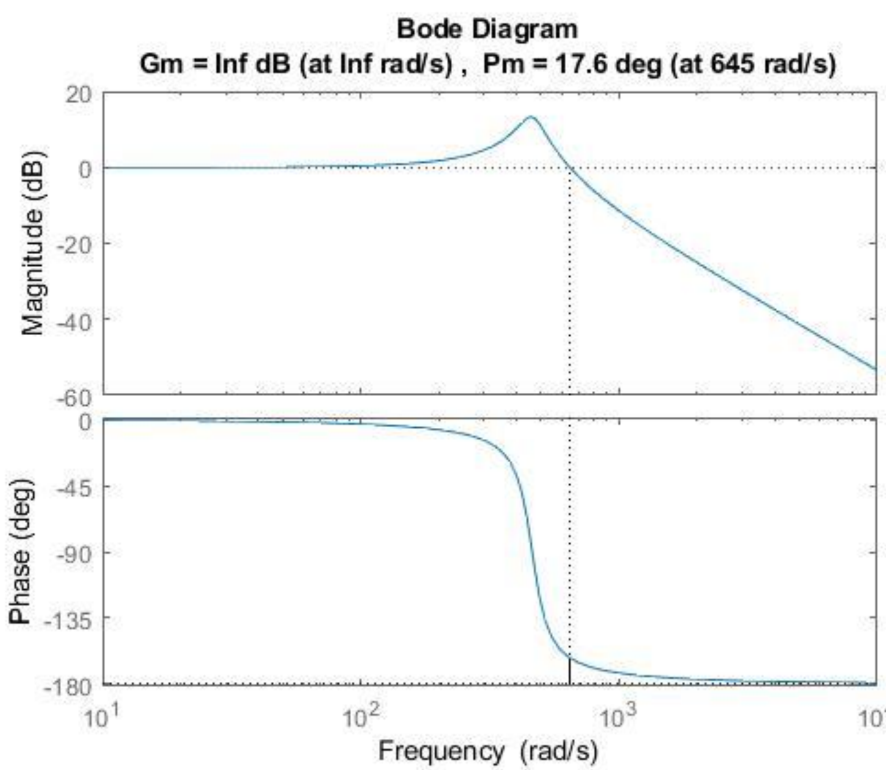


Figure 17: LRC Bode Plot with Phase Margins

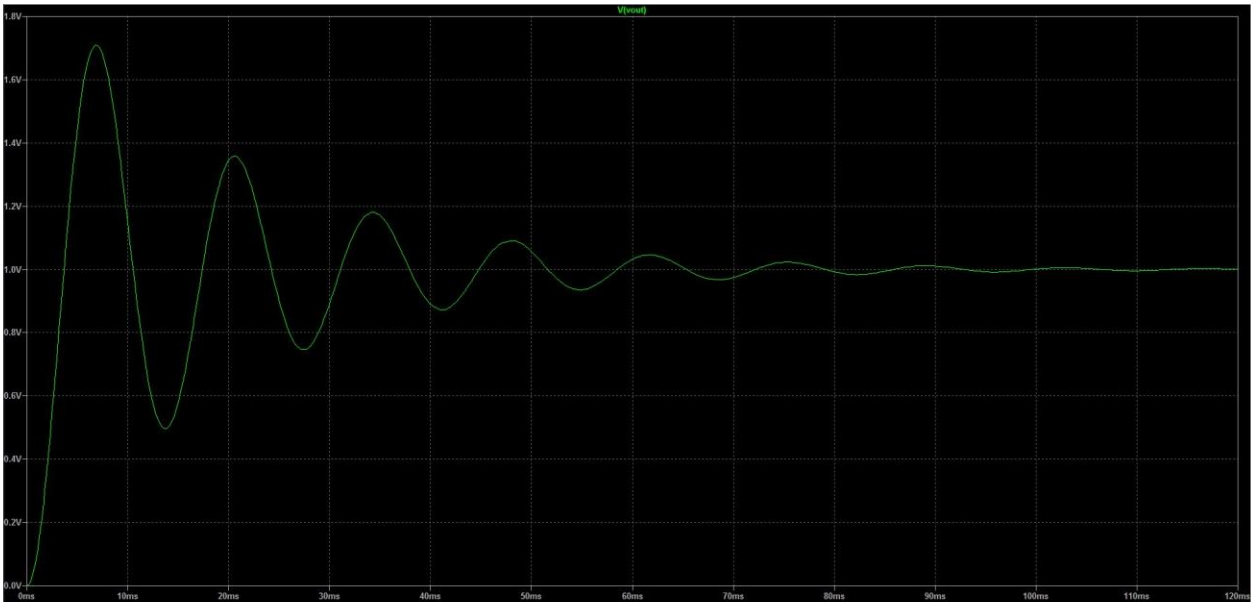


Figure 18: LRC LTSpice Simulation Step Response

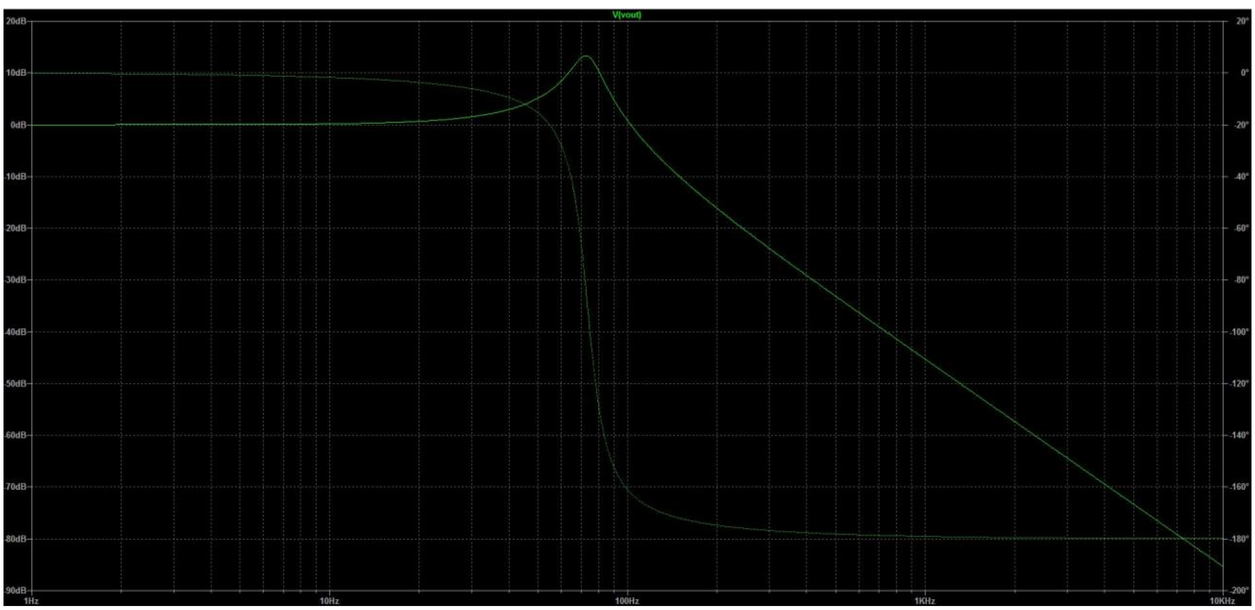


Figure 19: LRC LTSpice Simulation Bode Plot

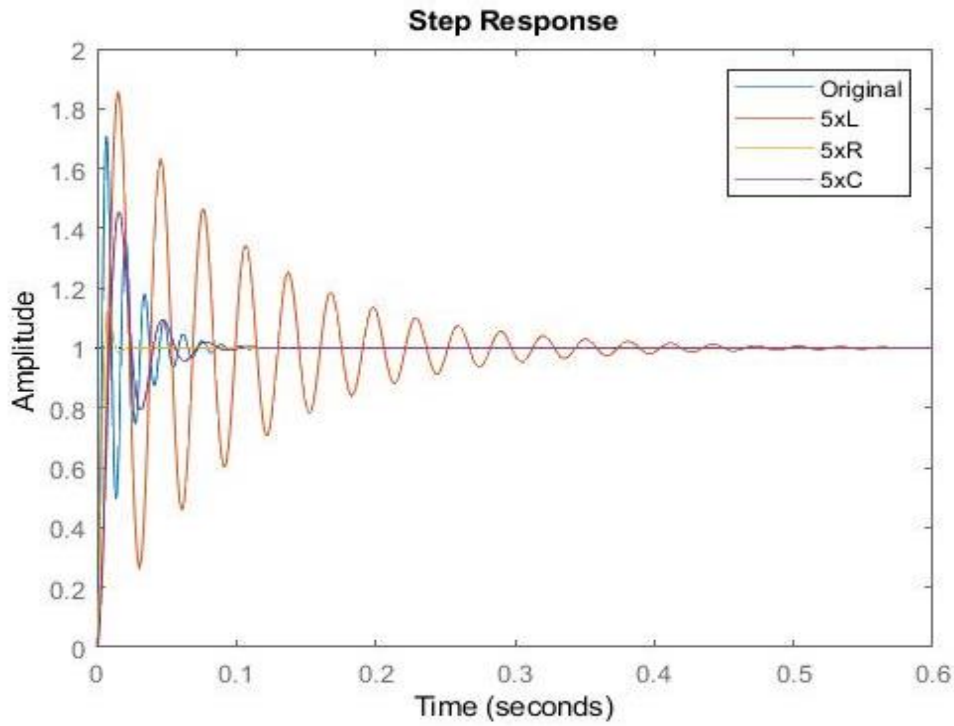


Figure 20: LRC 5times Component Increase Step response

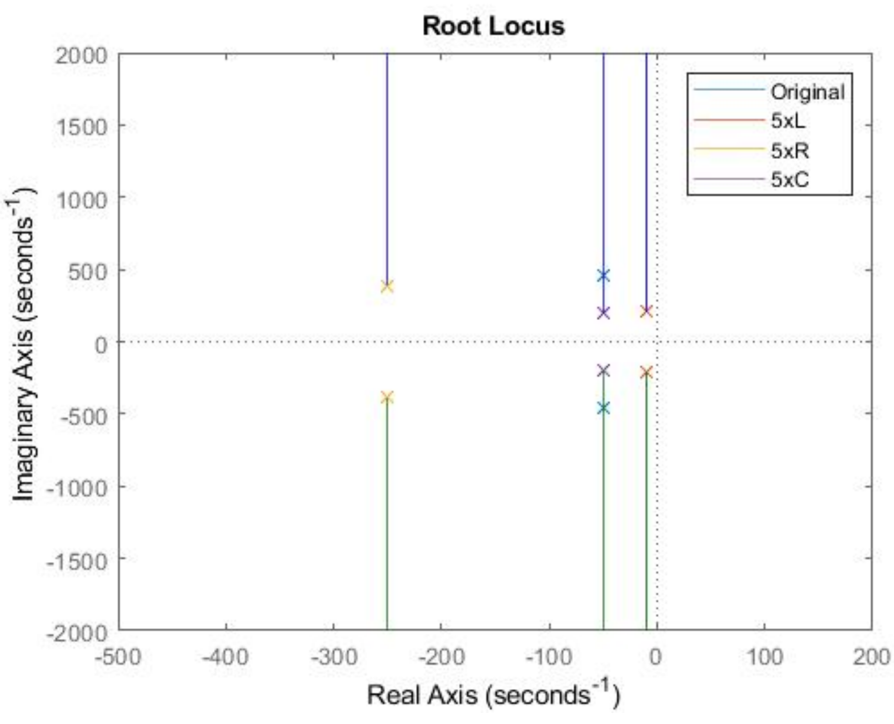


Figure 21: LRC 5times Component Increase Root Locus

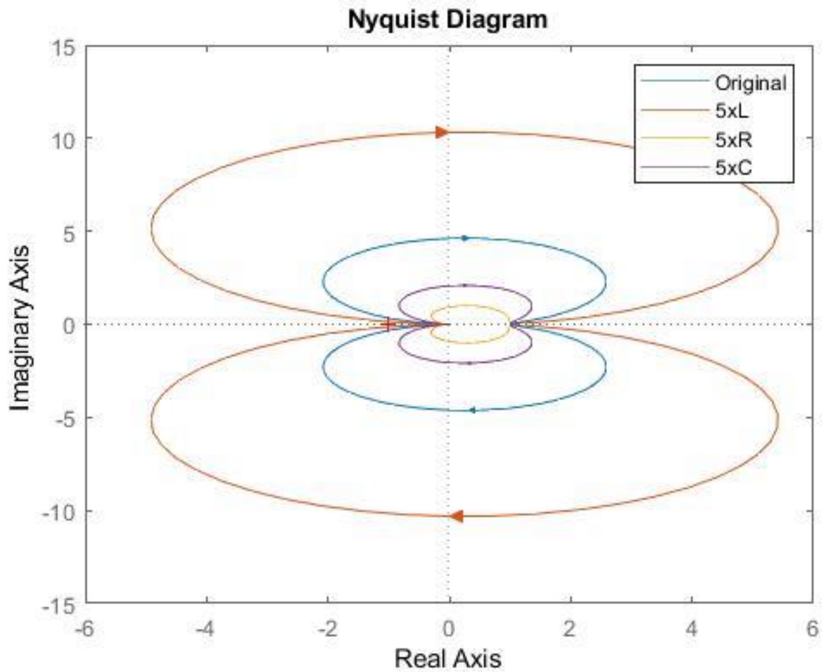


Figure 22: LRC 5times Component Increase Nyquist Plot

	Phase Margin
Original	17.6º
5 x Resistance	99.4º
5 x Inductance	7.86º
5 x Capacitance	40.1º

Table 1. LRC 5times Component Increase Phase Margin Difference

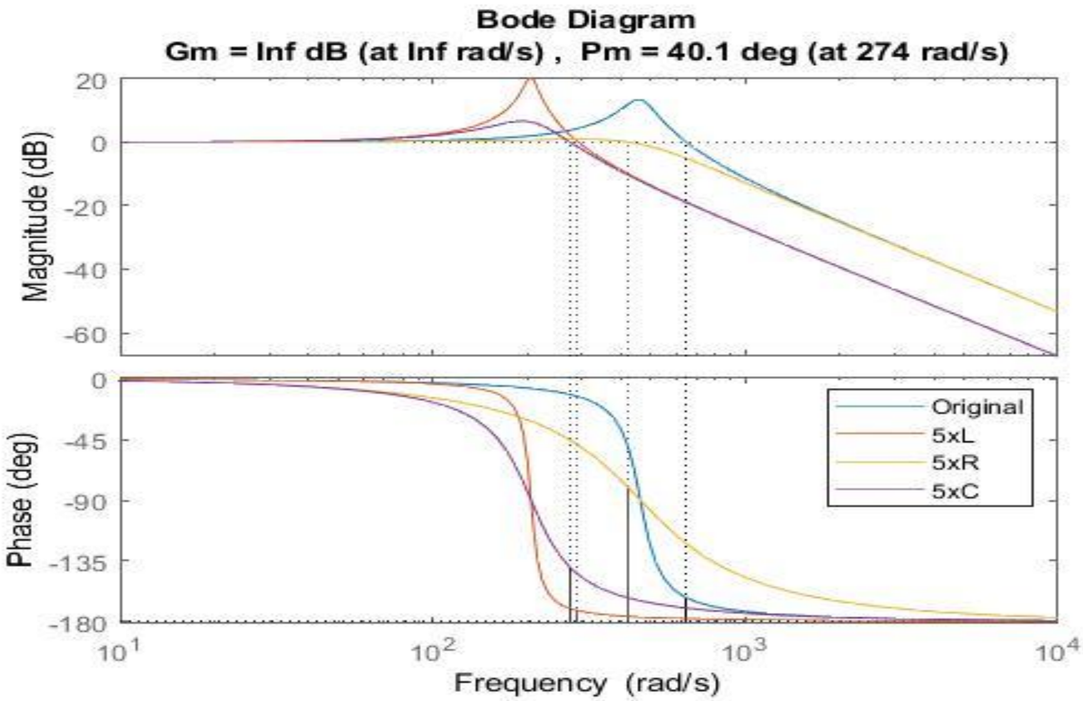


Figure 23: LRC 5times Component Increase Bode Plot

3.2 Negative Feedback

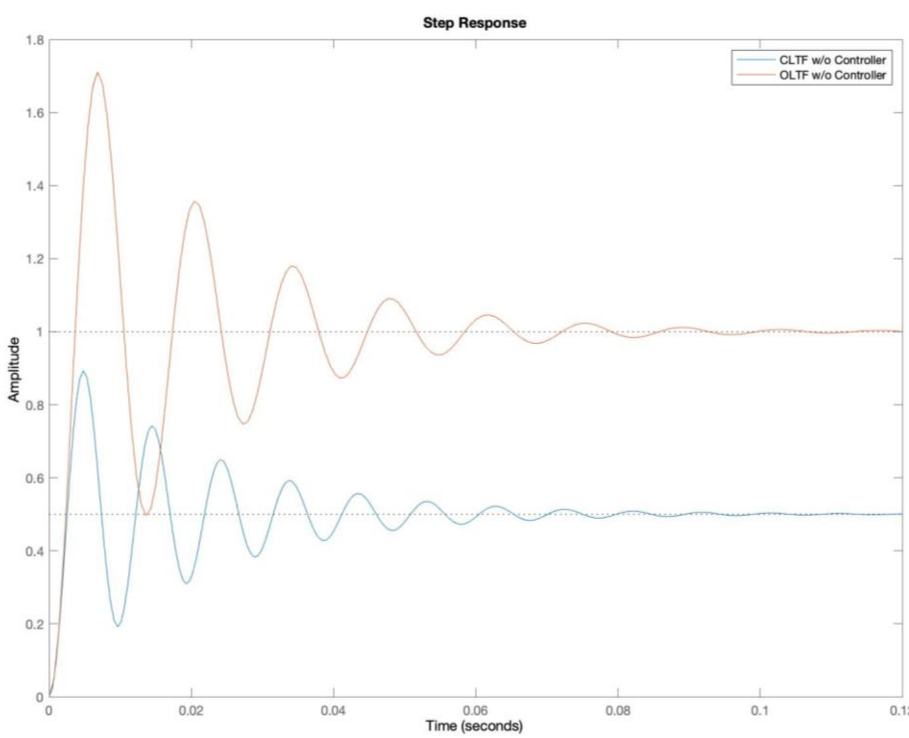


Figure 24: Open Loop & Closed Loop Step Response

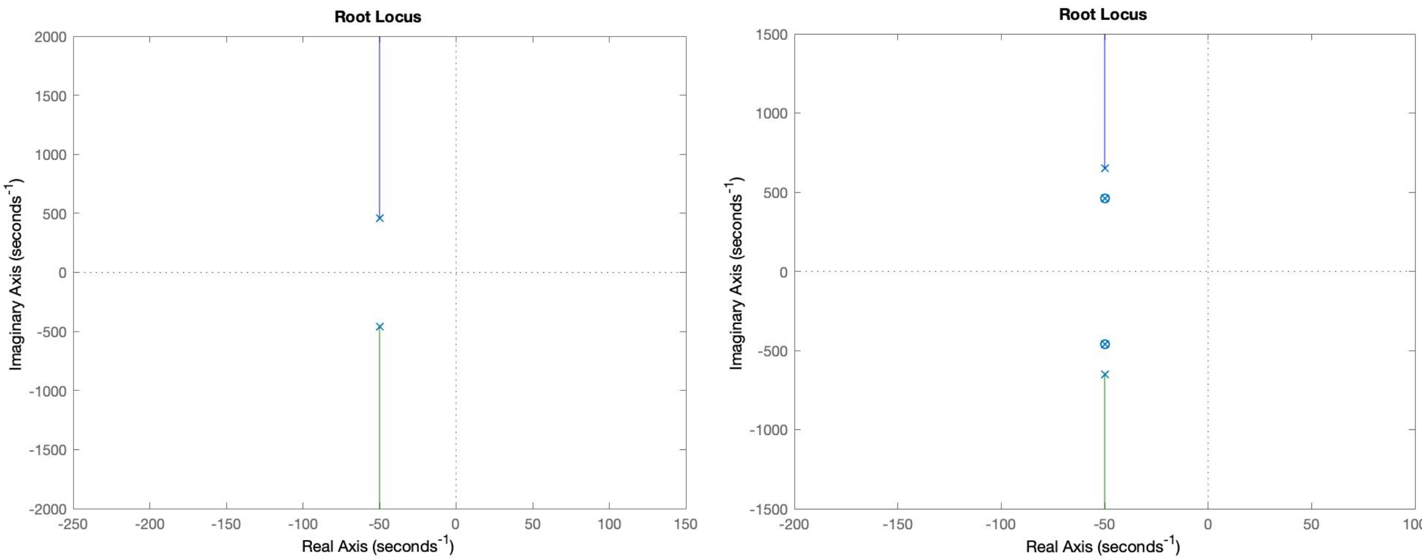


Figure 25: Root Locus (Left: Open-Loop, Right: Closed-Loop)

3.3 Controller Design

3.3.1 PID Control

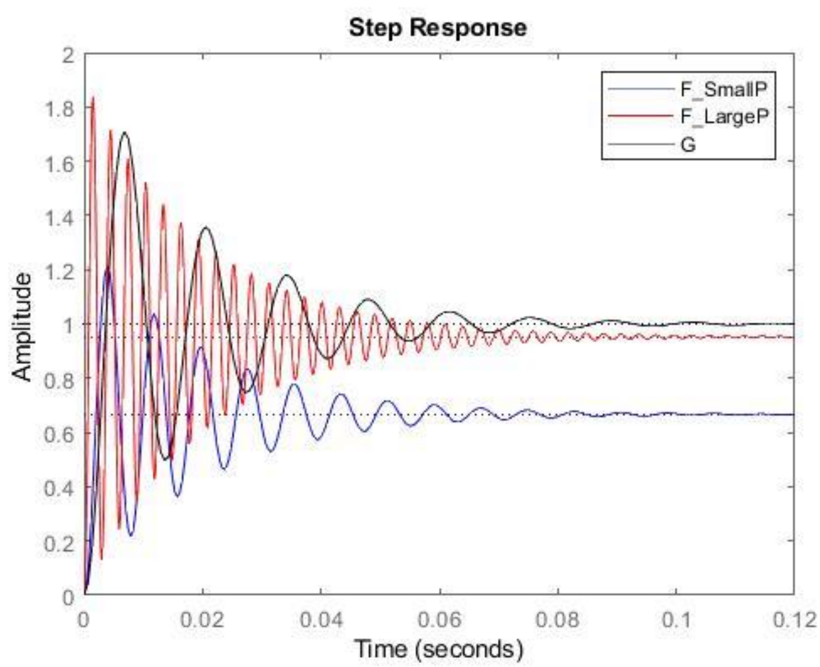


Figure 26: P Controller

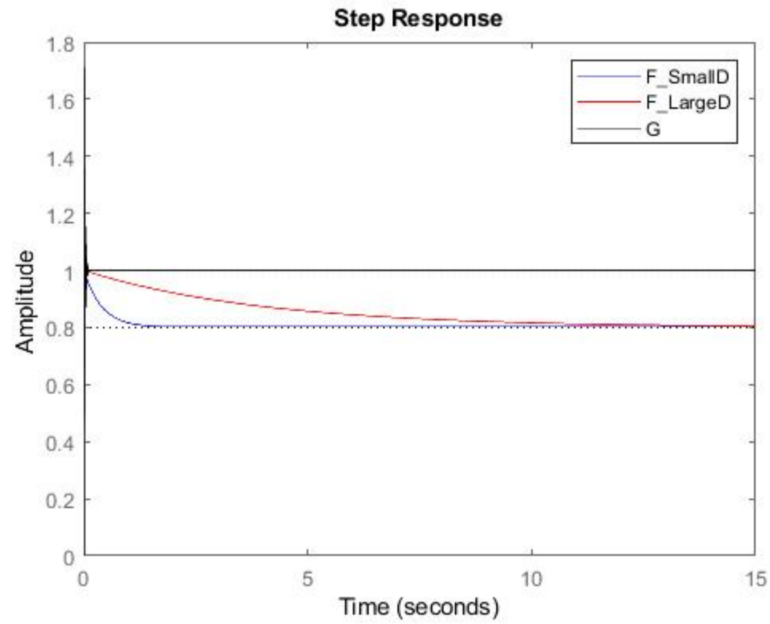


Figure 27: PD Controller

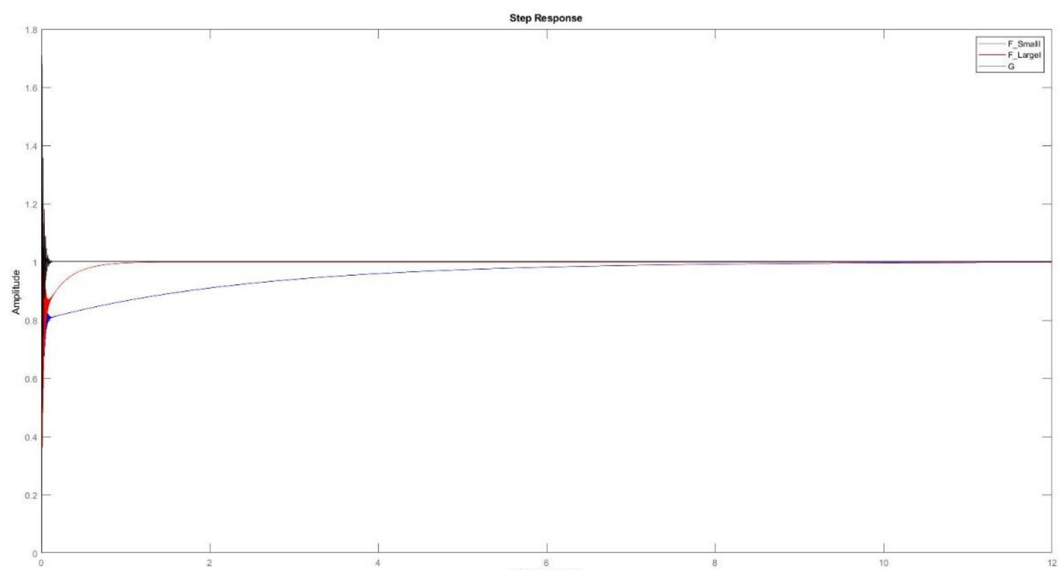


Figure 28: PI Controller

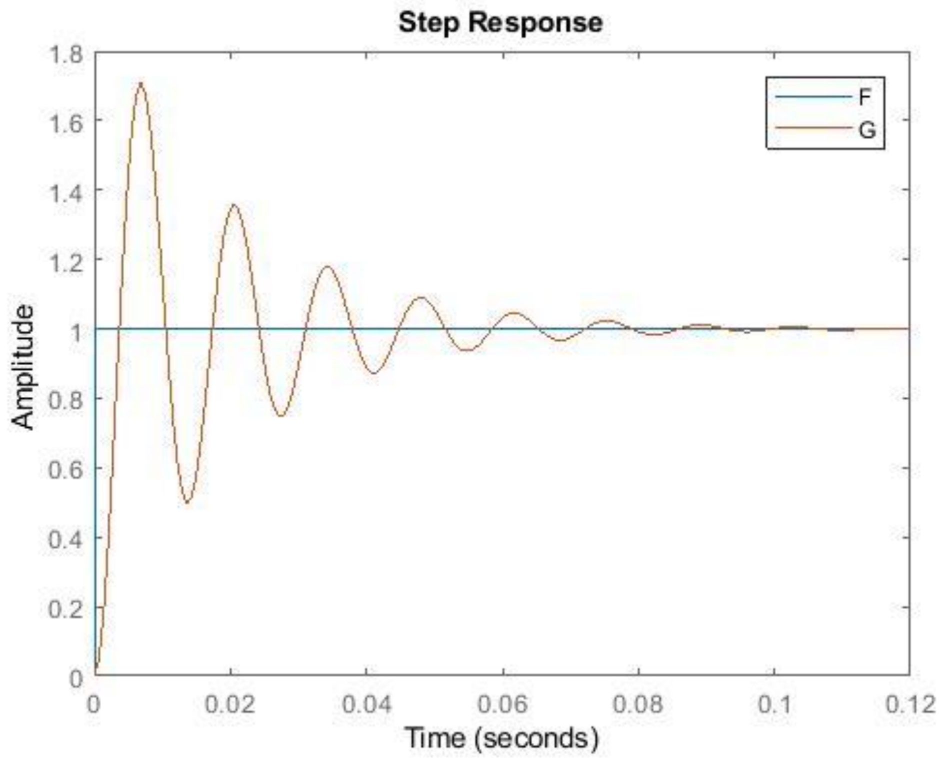


Figure 29: PID Step Response

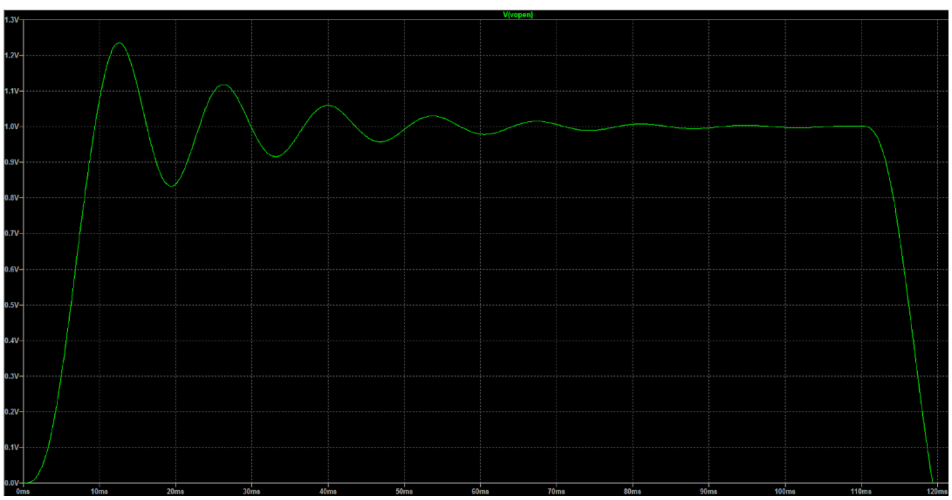


Figure 30: LTSpice PID Step Response

3.3.2 Lead-Lag Compensation

3.3.2.1 Lag Compensator Design

MATLAB Simulation

<u>Case I:</u> R1 = 1KΩ, R2=1KΩ, C2=1μF

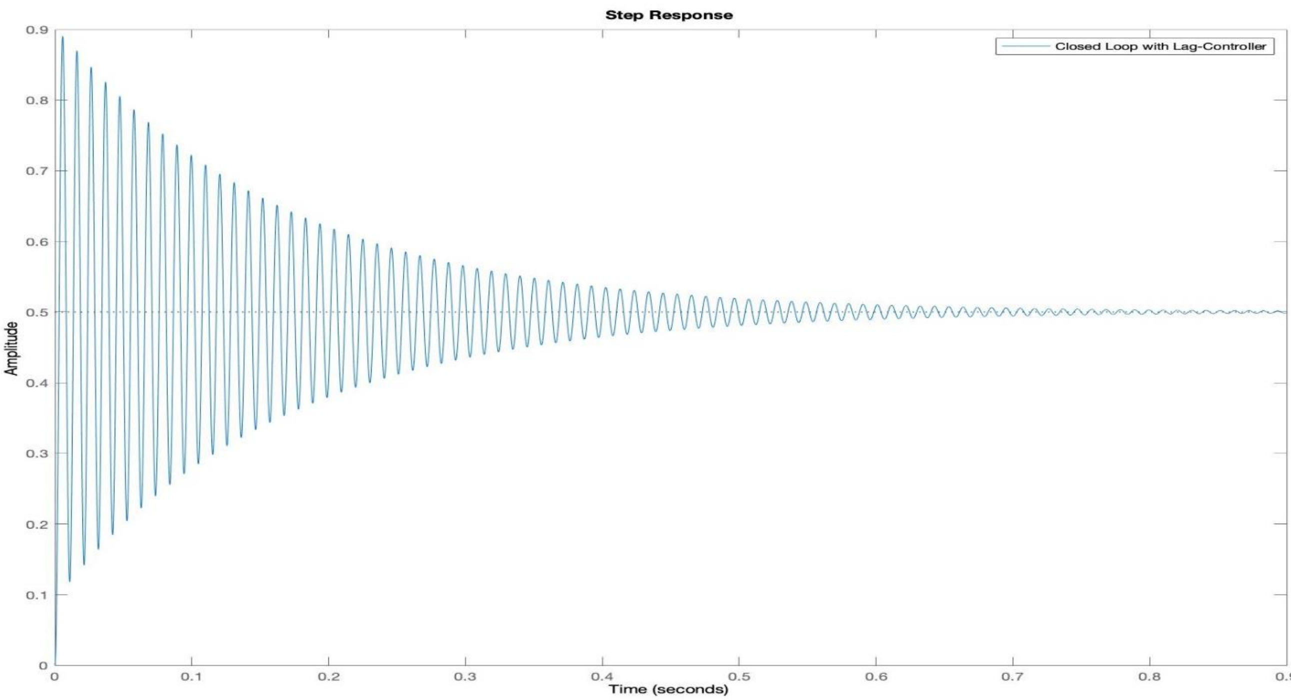


Figure 31: Step-Response of CL w/ Lag Compensator

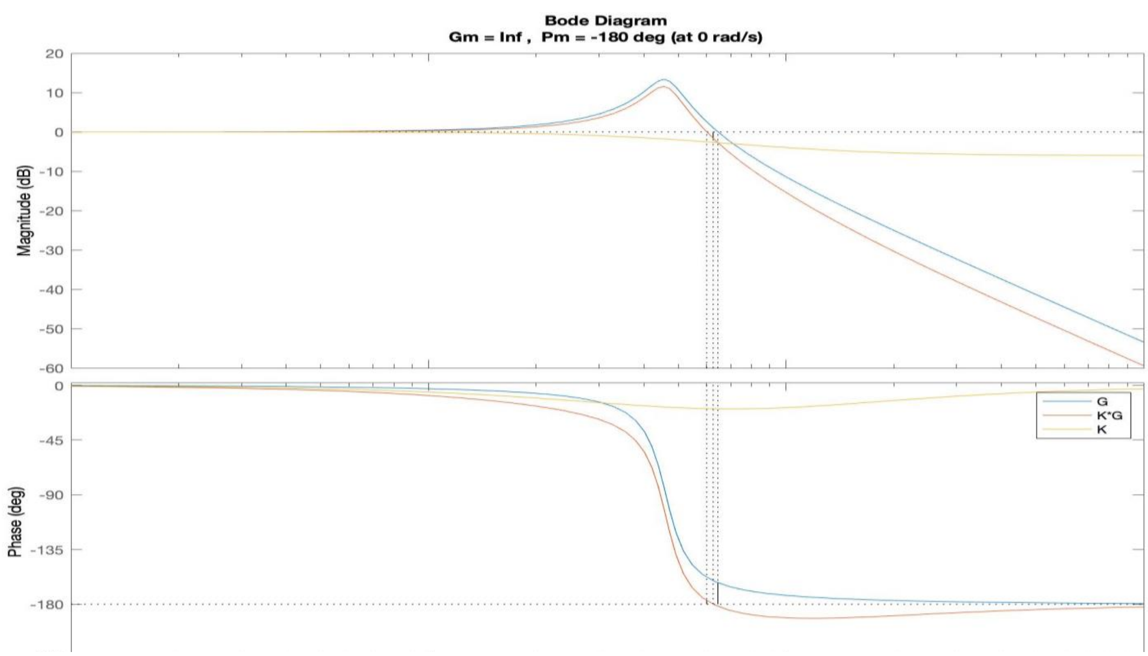


Figure 32: Bode Plot w/ Lag Compensator

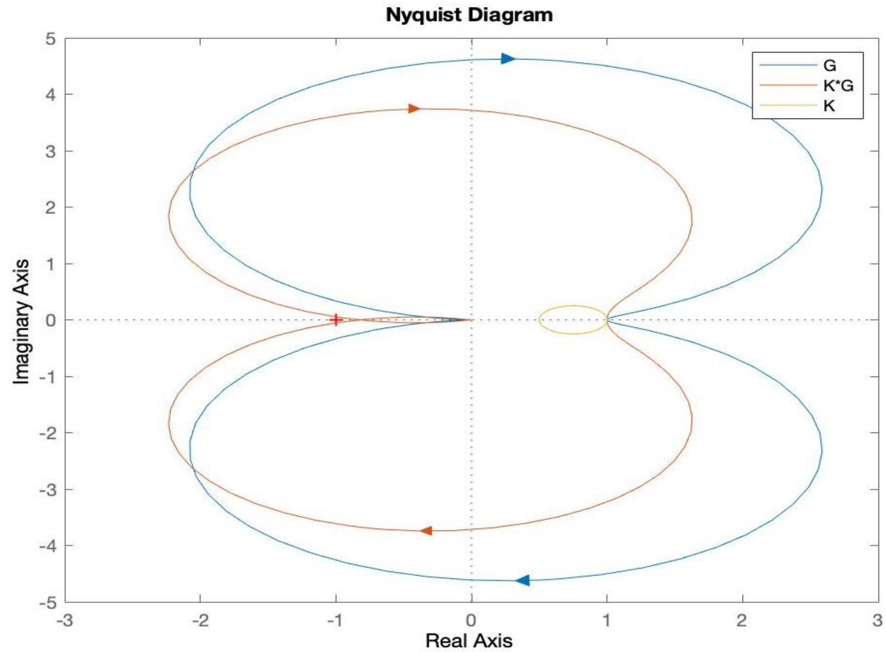


Figure 33: Nyquist Plot w/ Lag Compensator

<u>Case II:</u> R1 = 2KΩ, R2=1KΩ, C2=1μF

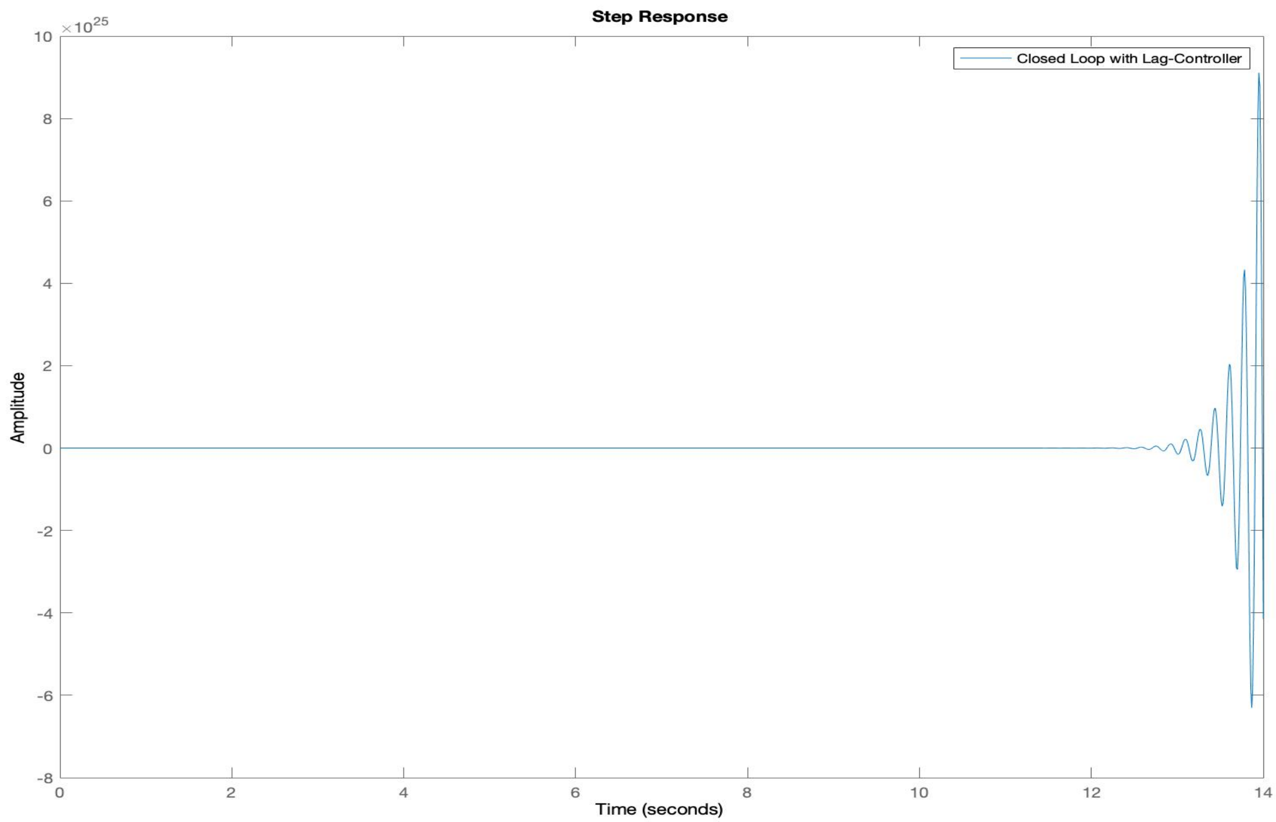


Figure 34: Increased R1 Closed Loop Lag Controller Step Response

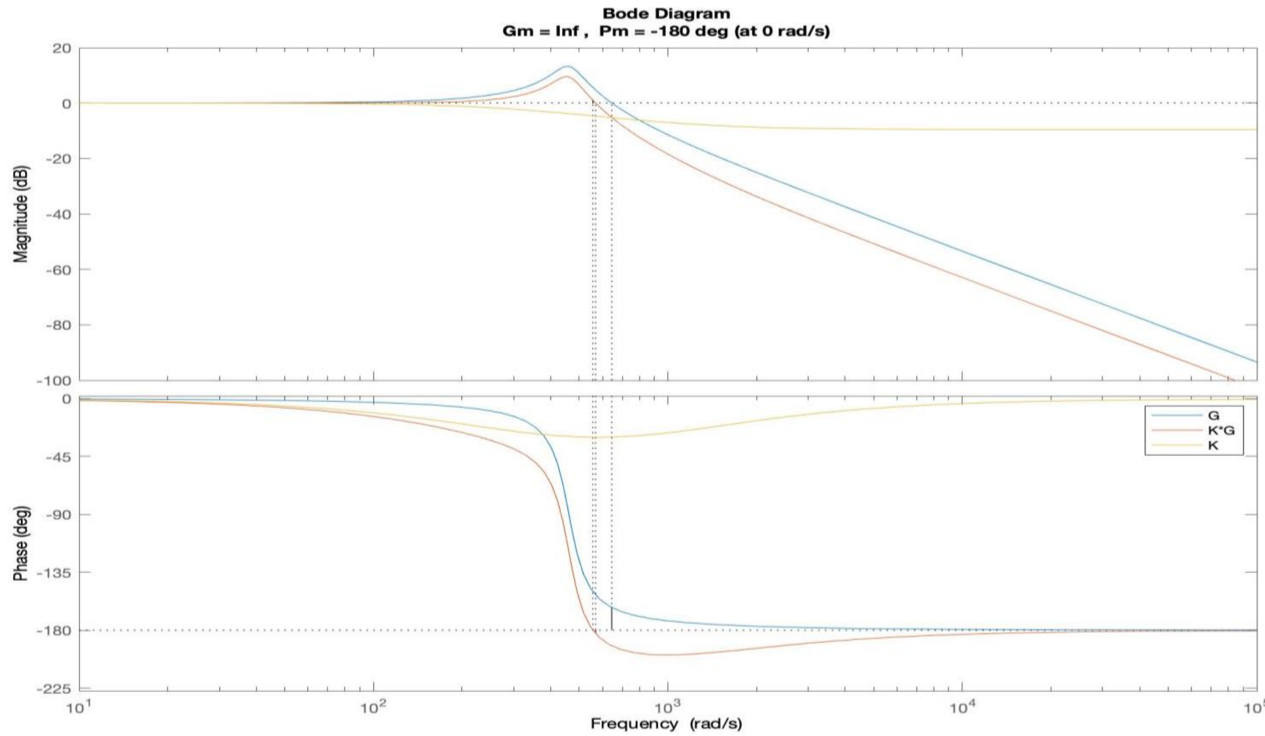


Figure 35: Increased R1 Lag Controller Bode Plot

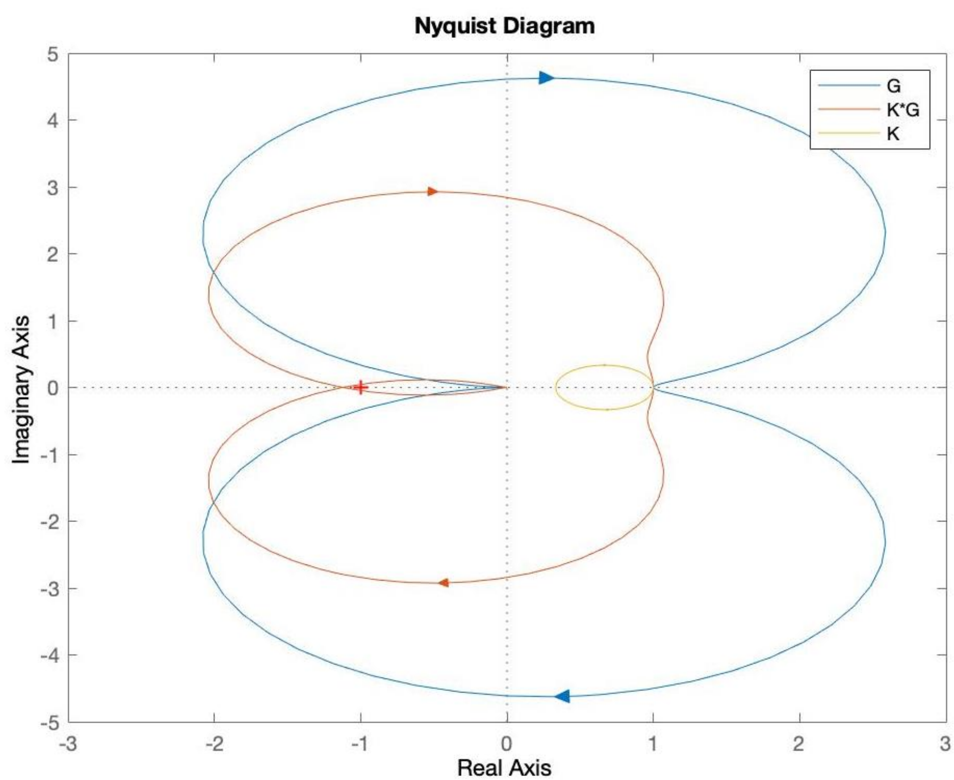


Figure 36: Increased R1 Lag Controller Nyquist Plot

LTSpice Simulation

Case I: R1 = 1KΩ, R2=1KΩ, C2=1µF

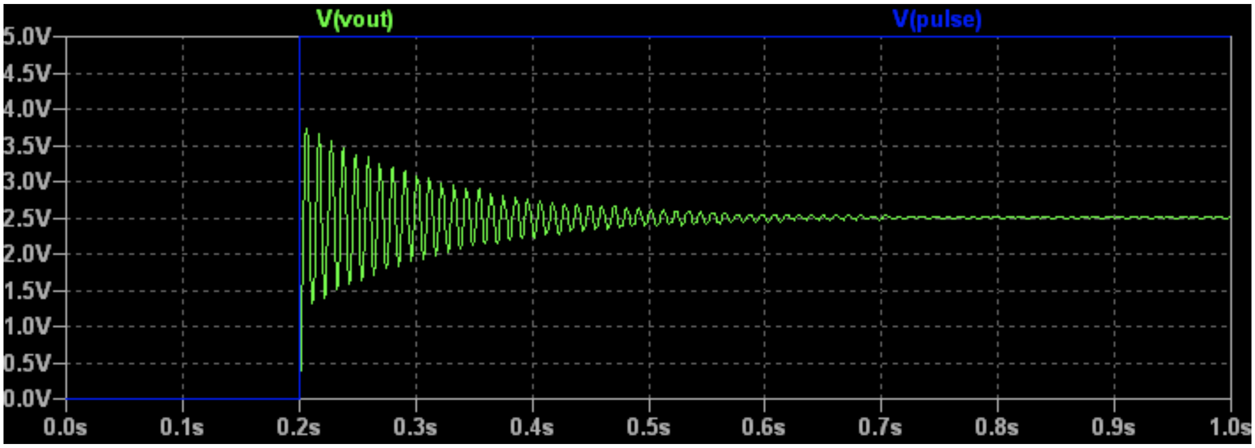


Figure 37: LTSpice Lag controller Step Response

Case II: R1 = 2KΩ, R2=1KΩ, C2=1µF

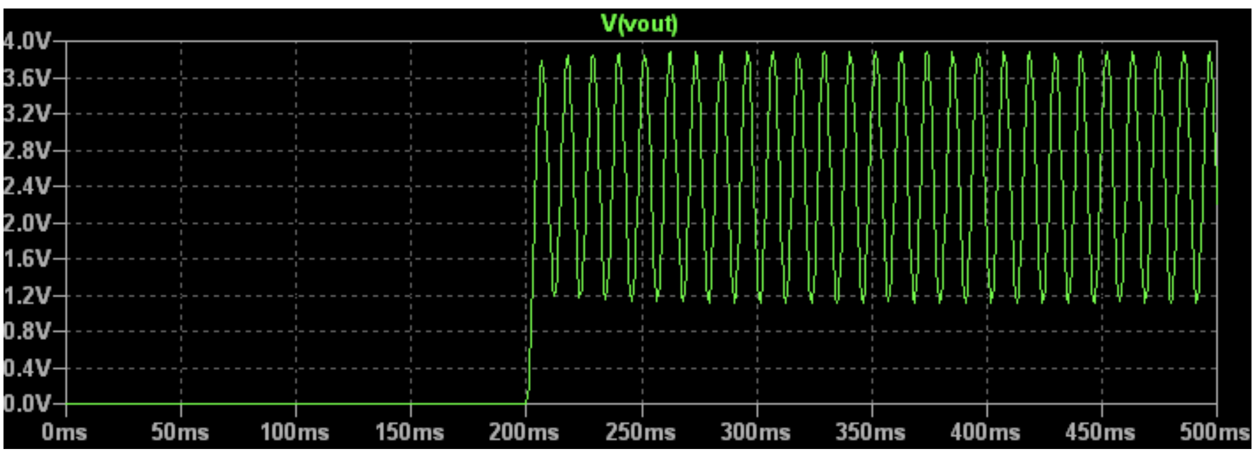


Figure 38: LTSpice Increased R1 Lag Controller Step Response

3.3.2.2 Lead Compensator Design

MATLAB Simulation

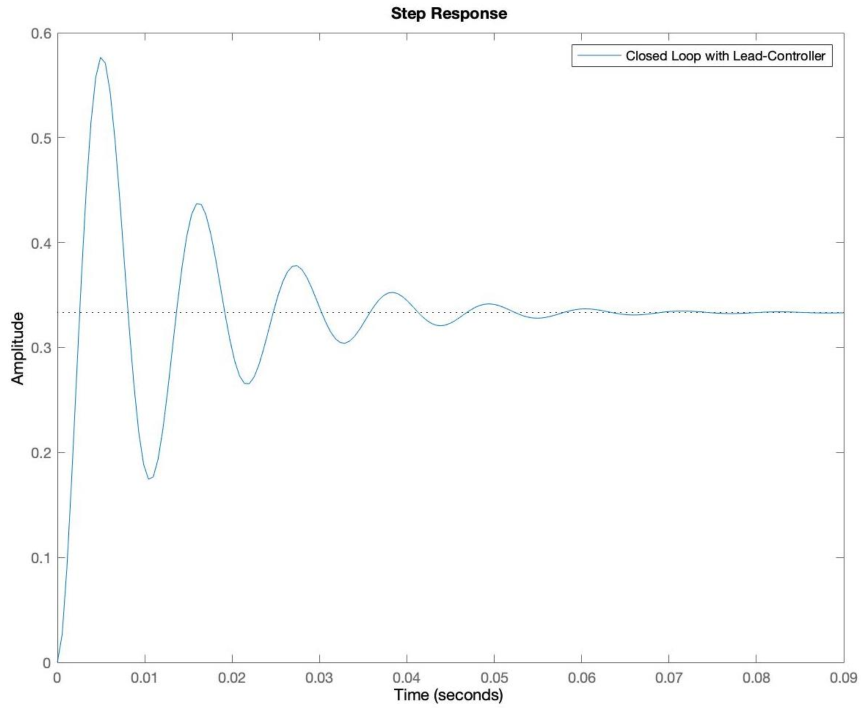


Figure 39: Closed Loop Lead Controller Step Response

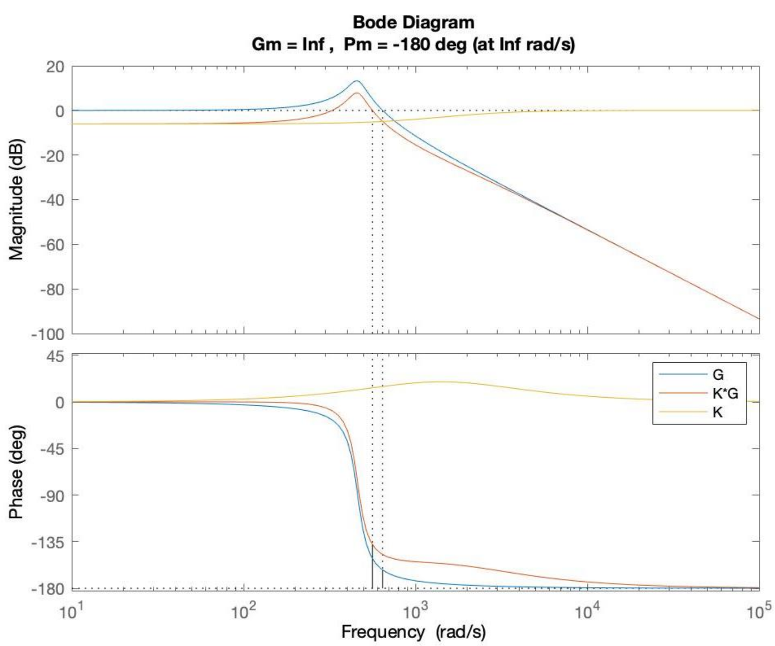


Figure 40: Lead Controller Bode Plot

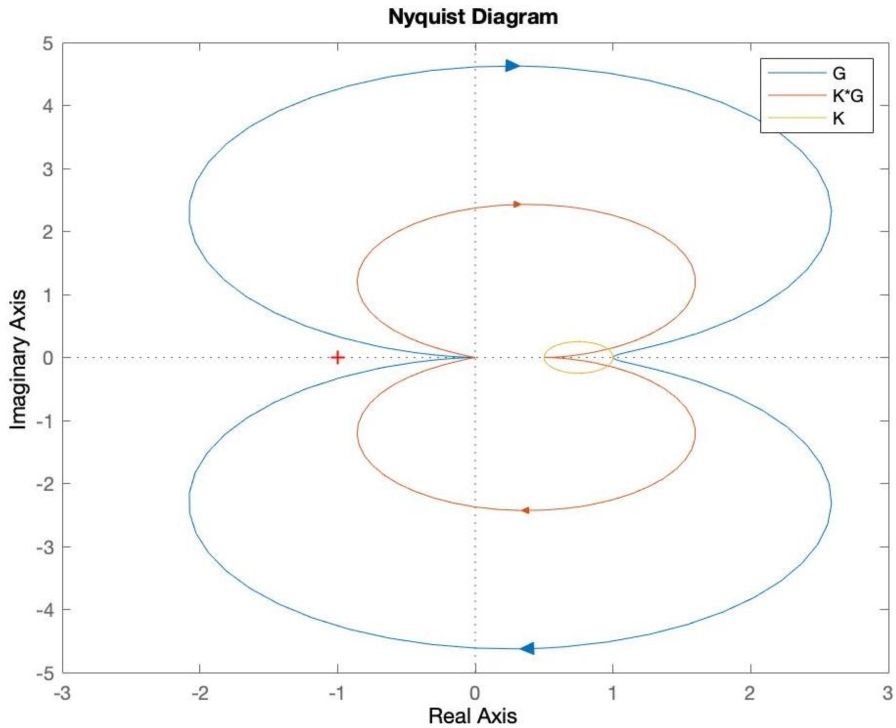


Figure 41: Lead Controller Nyquist Plot

3.3.2.3 Lead-Lag Compensator Design

MATLAB Simulation

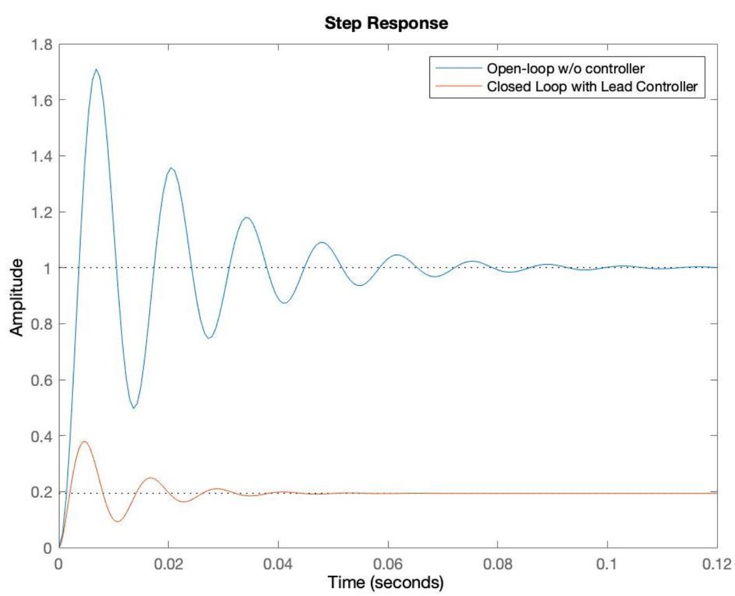


Figure 42: Step Response w/ Lead Compensator at OL PM frequency

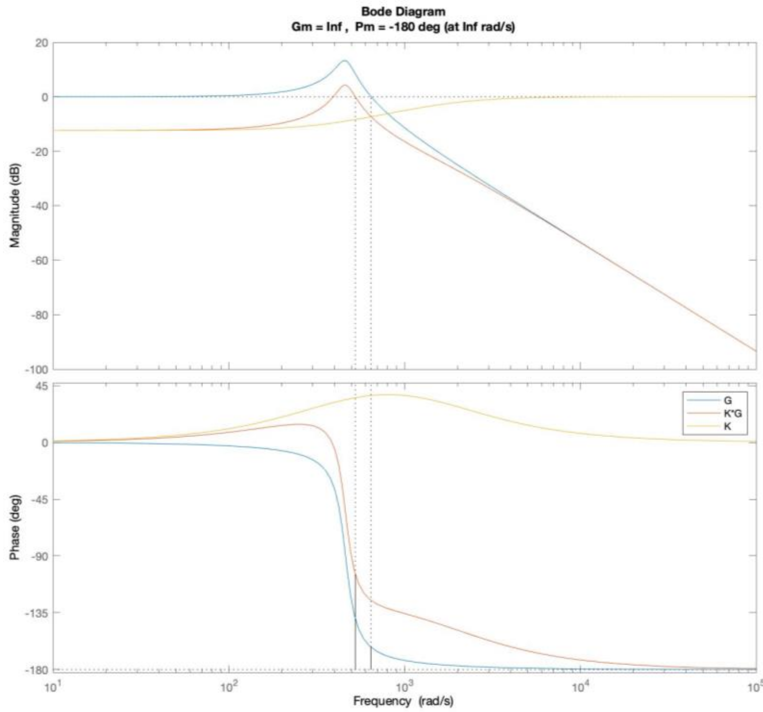


Figure 43: Bode Plot

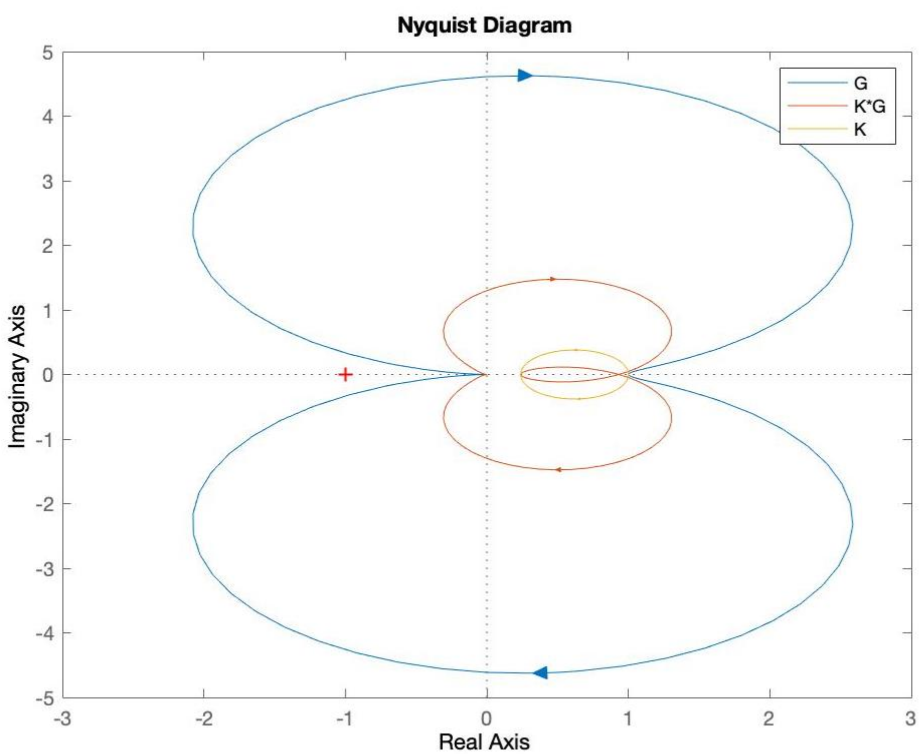


Figure 44: Nyquist Plot

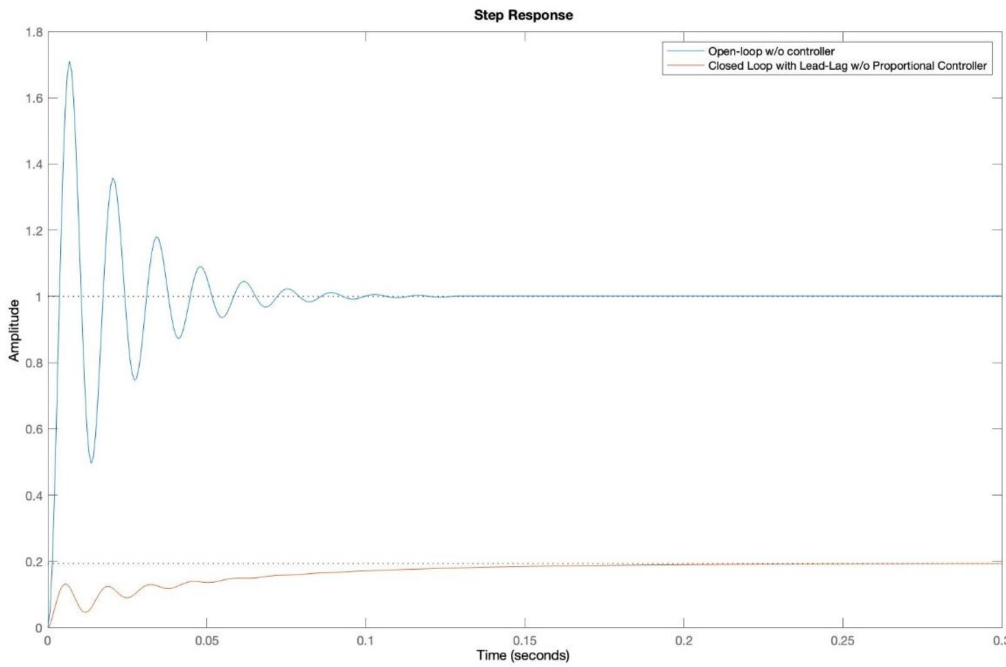


Figure 45: Lead-Lag Controller Step Response

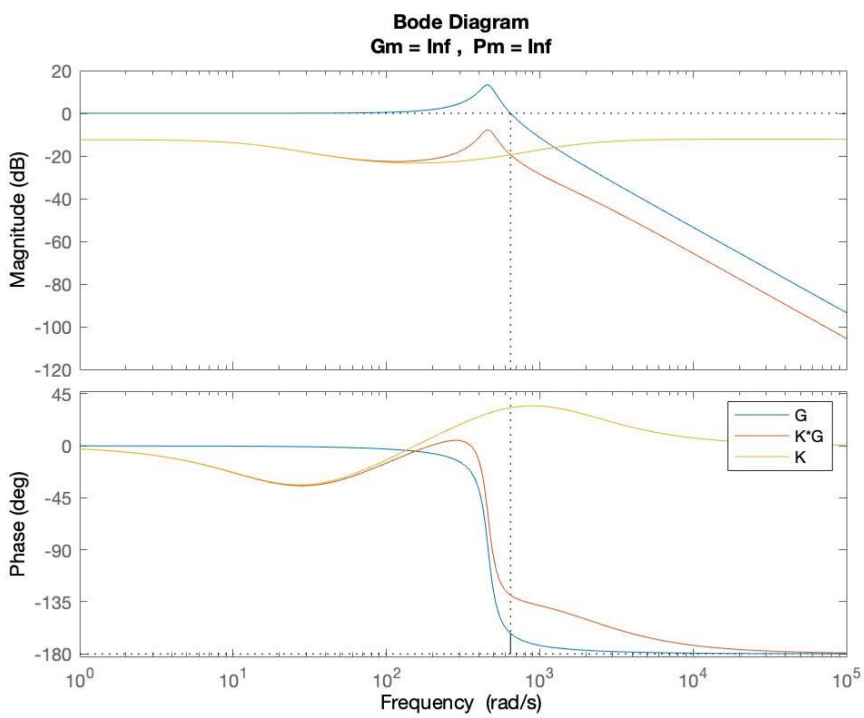


Figure 46: Lead-Lag Controller Bode Plot

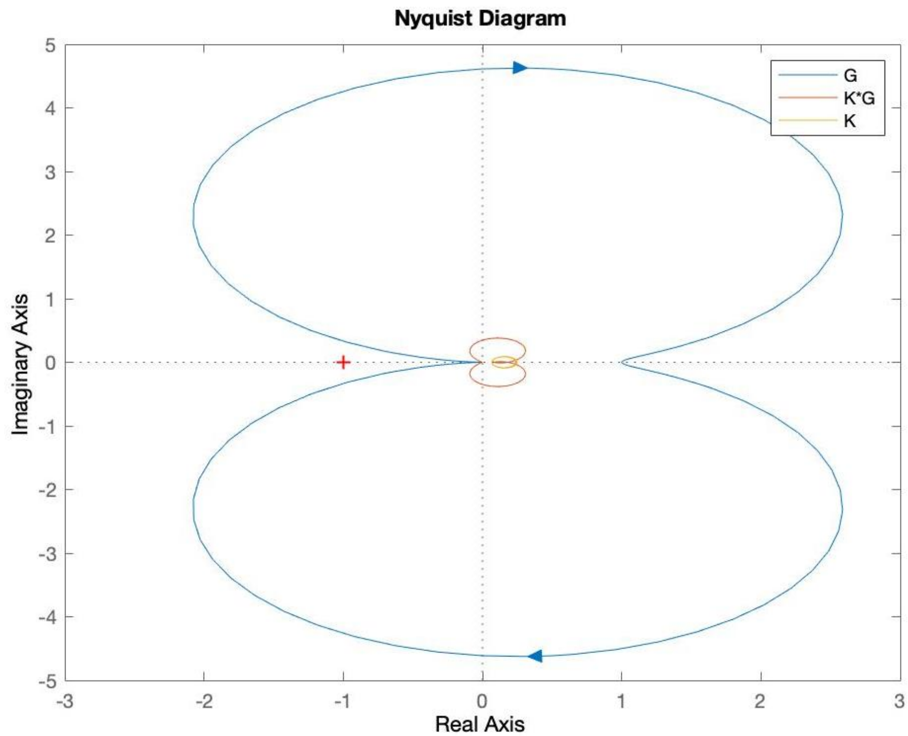


Figure 47: Lead-Lag Controller Nyquist Plot

Adding Kp

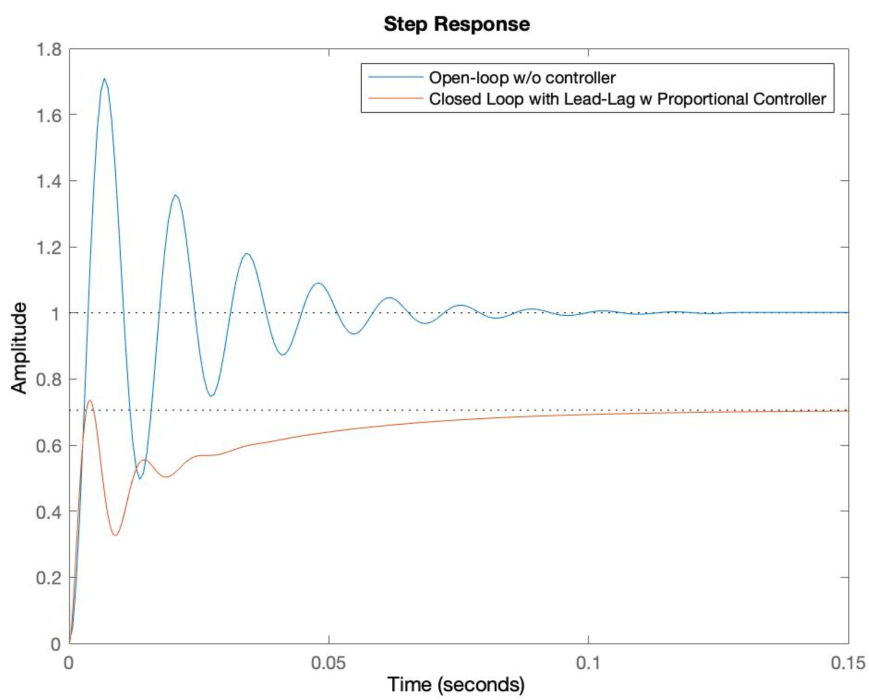


Figure 48: Step Response w/ Lead-Lag Controller w/ Proportional Gain of 10

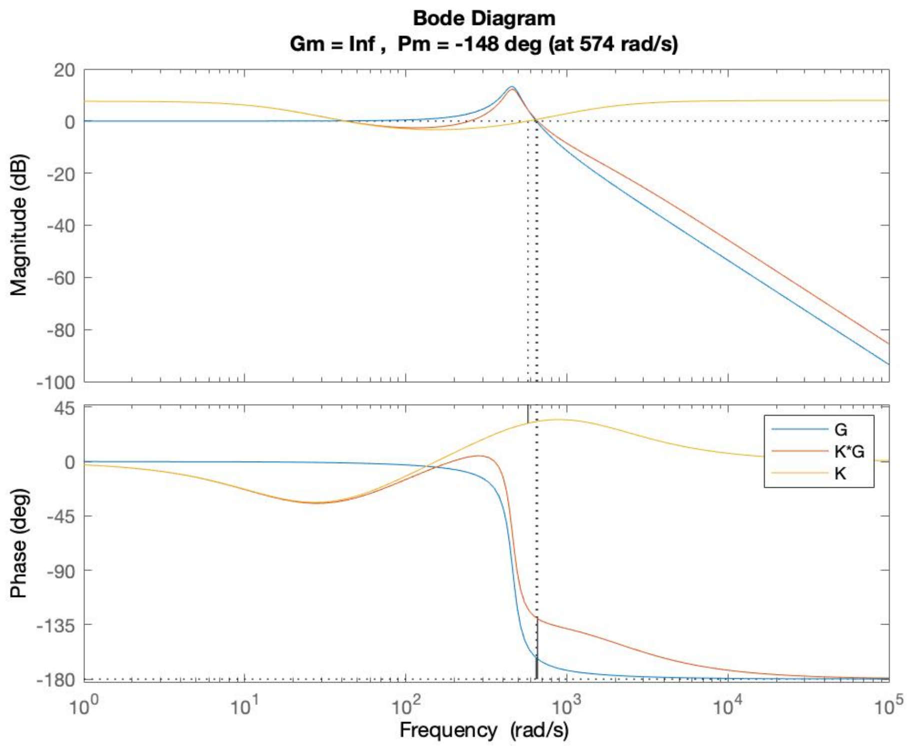


Figure 49: Bode plot of Lead-Lag Controller w/ Proportional Gain of 10

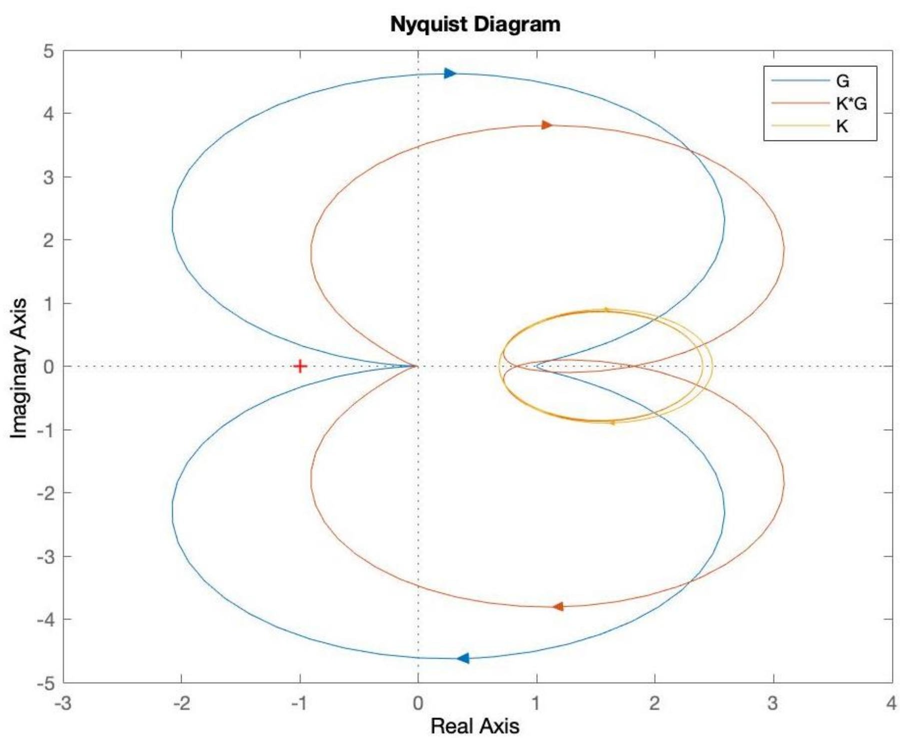


Figure 50: Nyquist plot of Lead-Lag Controller w/ Proportional Gain of 10

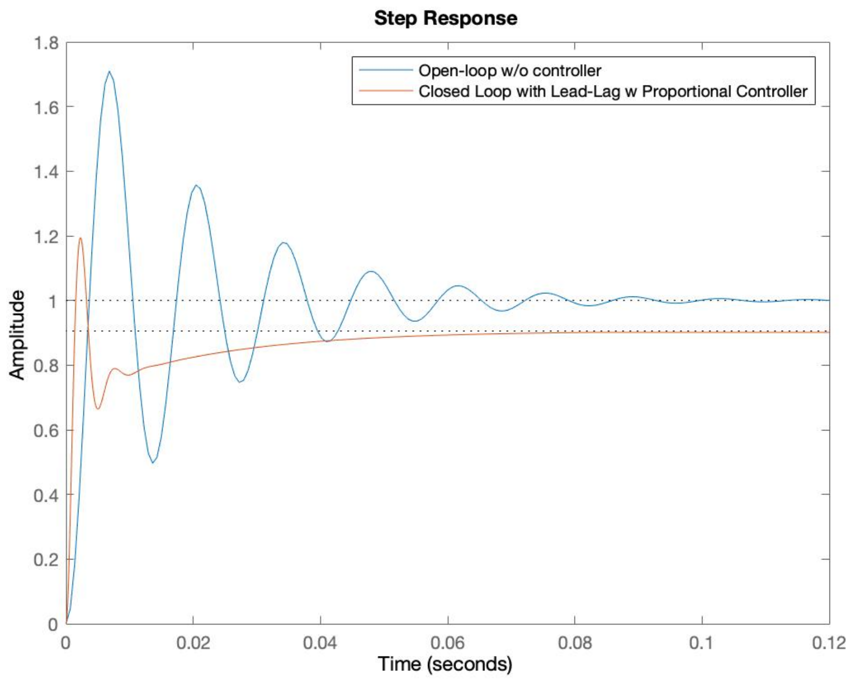


Figure 51: Step-response of Lead-Lag Controller w/ Proportional Gain of 40

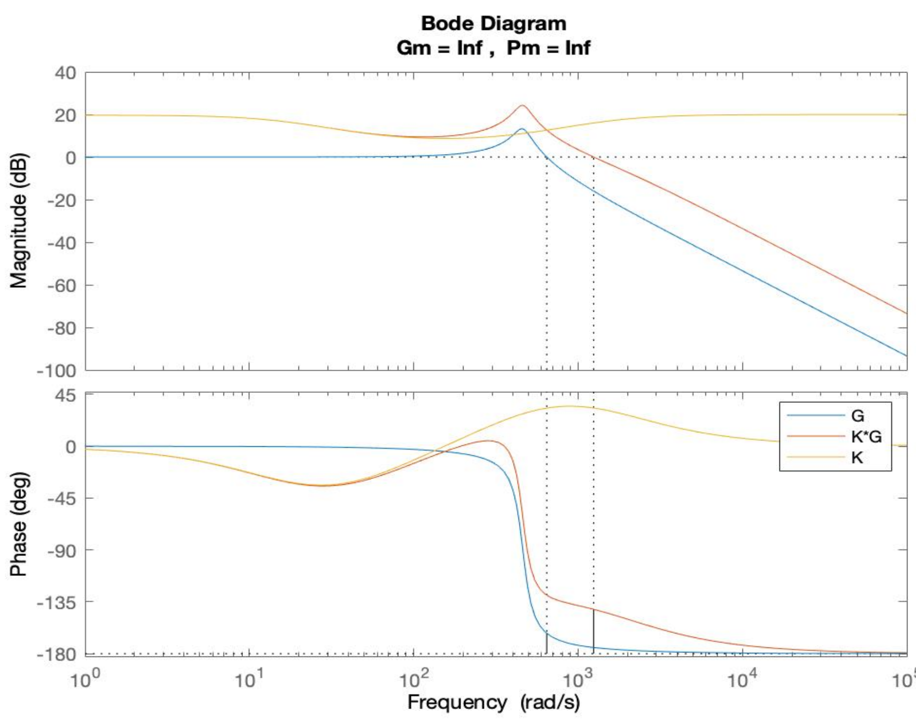


Figure 52: Bode plot of Lead-Lag Controller w/ Proportional Gain of 40

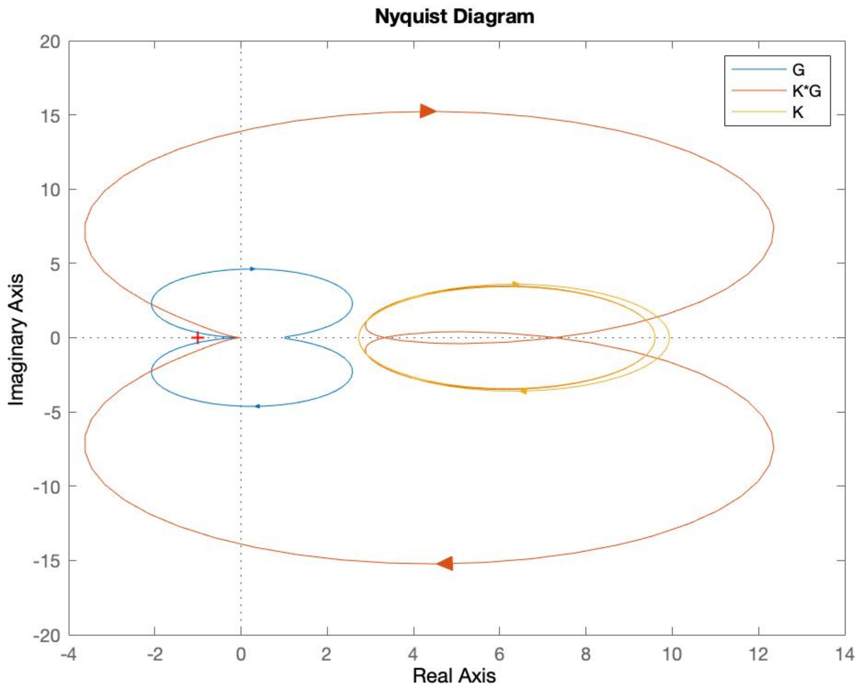


Figure 53: Nyquist plot of Lead-Lag Controller w/ Proportional Gain of 40

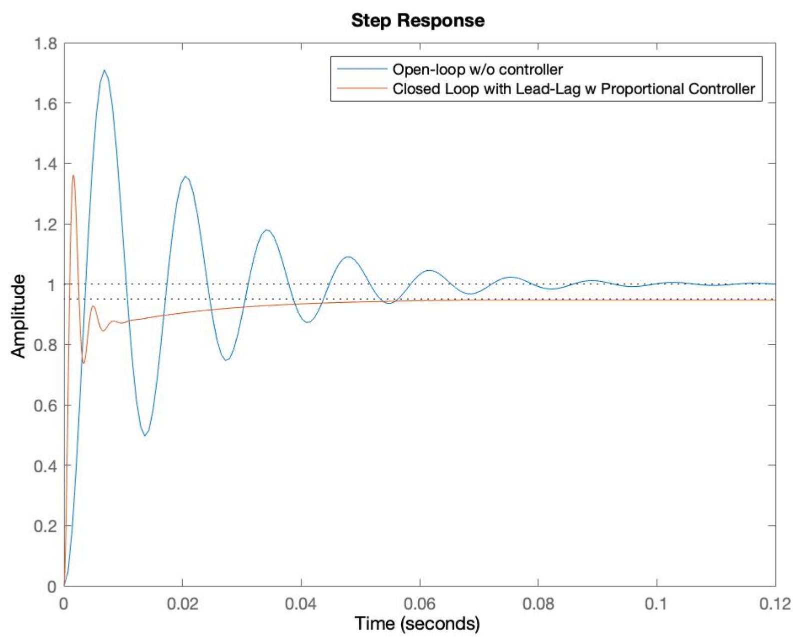


Figure 54: Step Response of Lead-Lag Controller w/ Proportional Gain 80

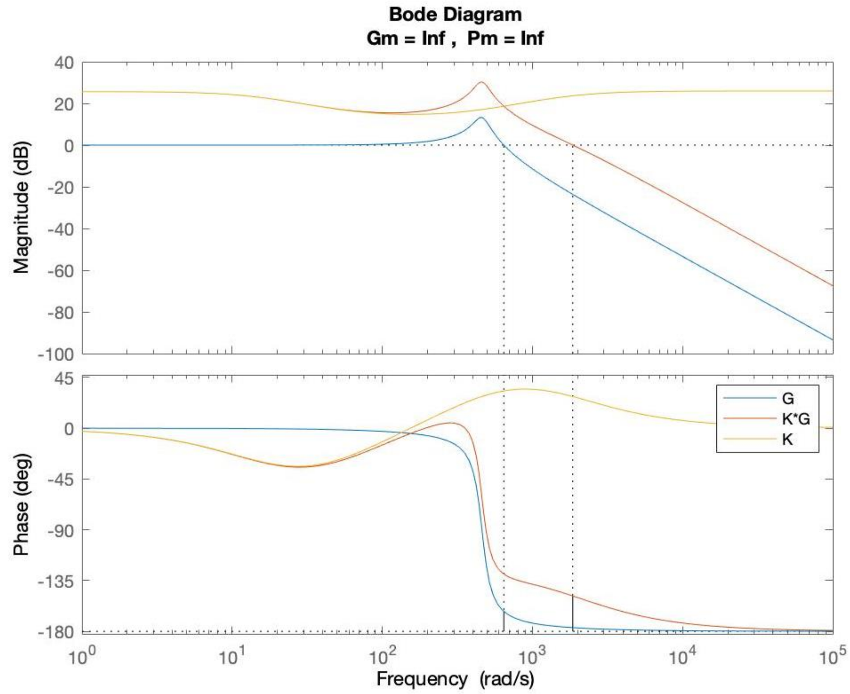


Figure 55: Bode plot of Lead-Lag Controller w/ Proportional Gain of 80

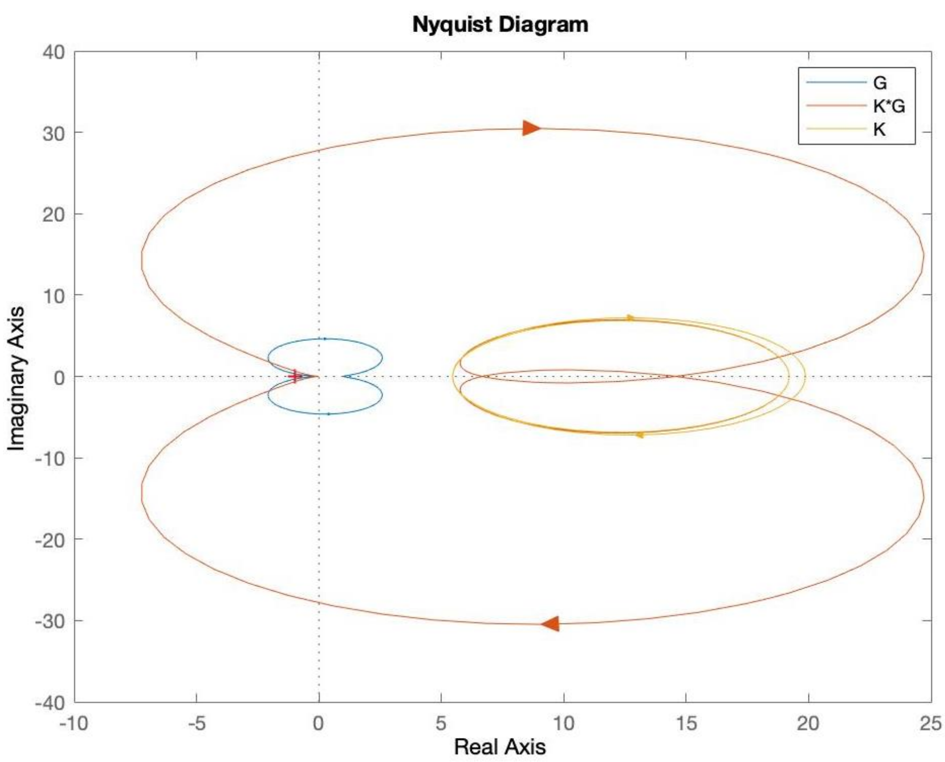


Figure 56: Nyquist plot of Lead-Lag Controller w/ Proportional Gain 80

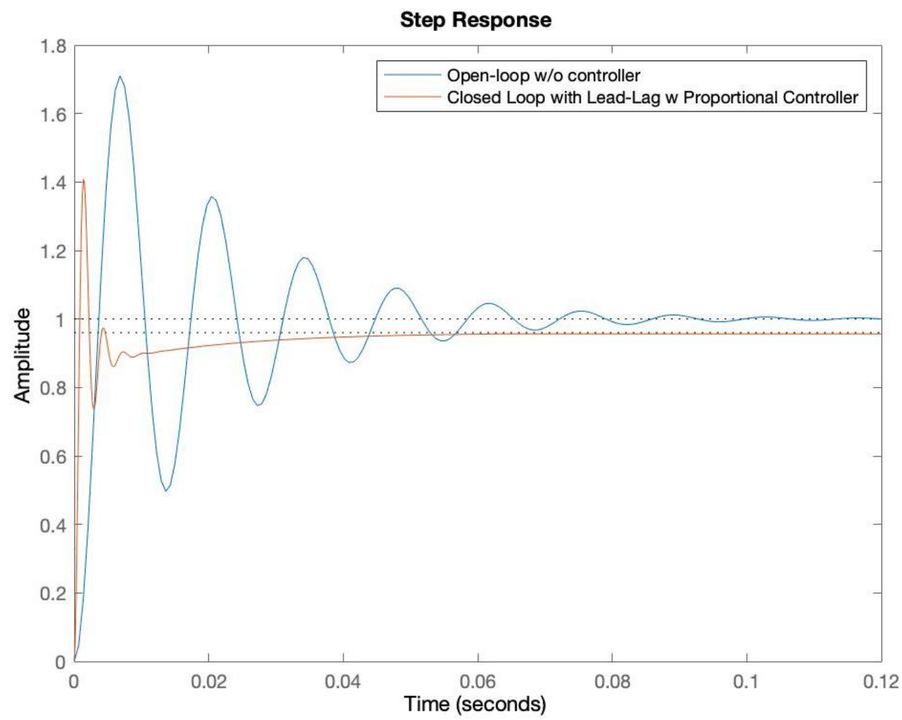


Figure 57: Step Response of Lead-Lag Controller w/ Proportional Gain of 100

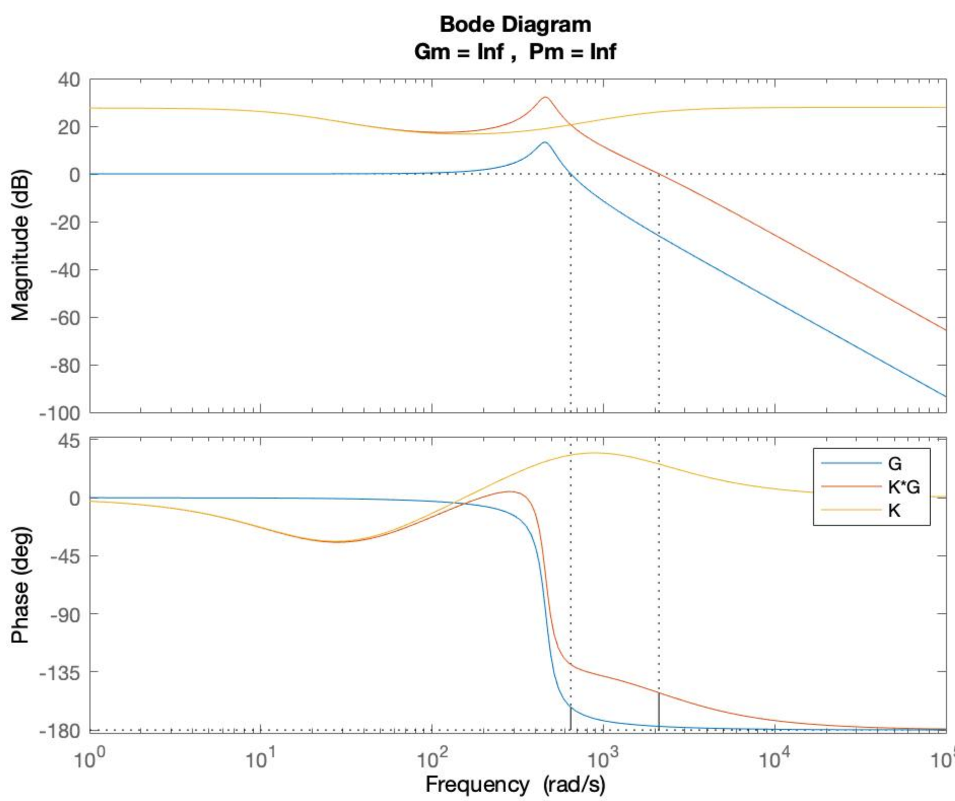


Figure 58: Bode plot of Lead-Lag Controller w/ Proportional Gain of 100

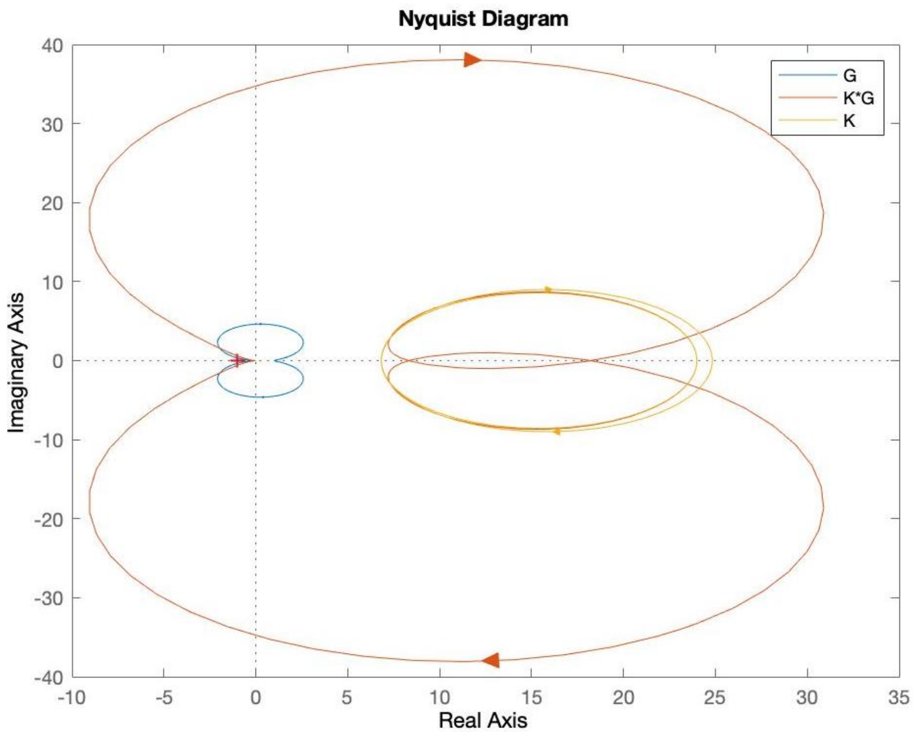


Figure 59: Nyquist plot of Lead-Lag Controller w/ Proportional Gain of 100

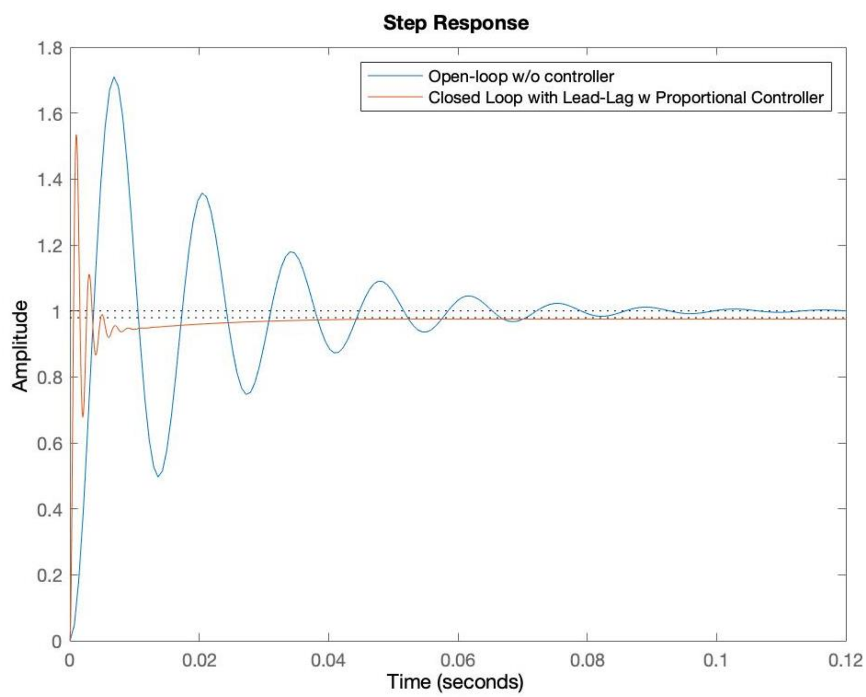


Figure 60: Step-response of Lead-Lag Controller w/ Proportional Gain of 200

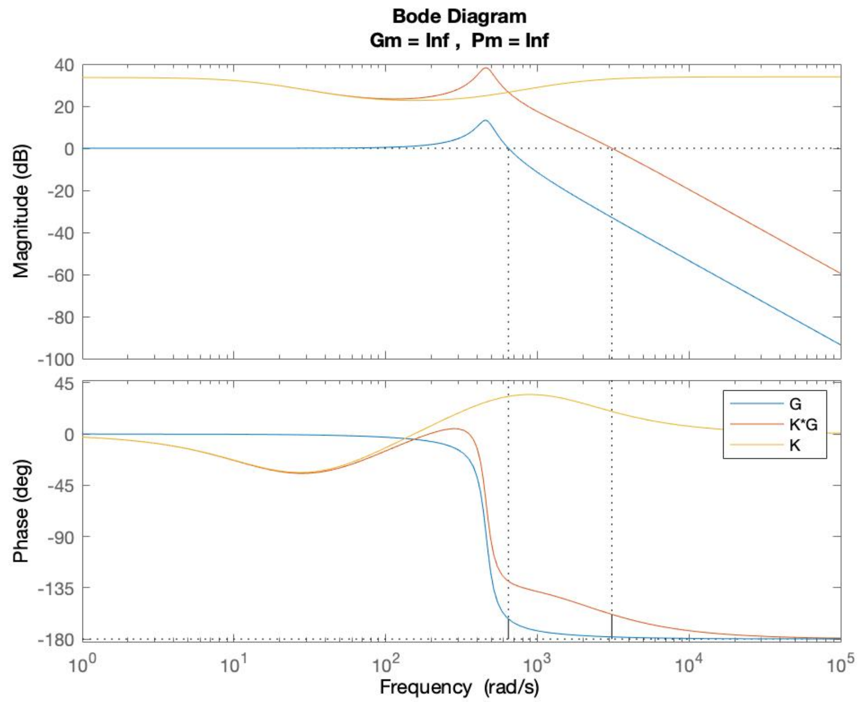


Figure 61: Bode plot of Lead-Lag Controller w/ Proportional Gain of 200

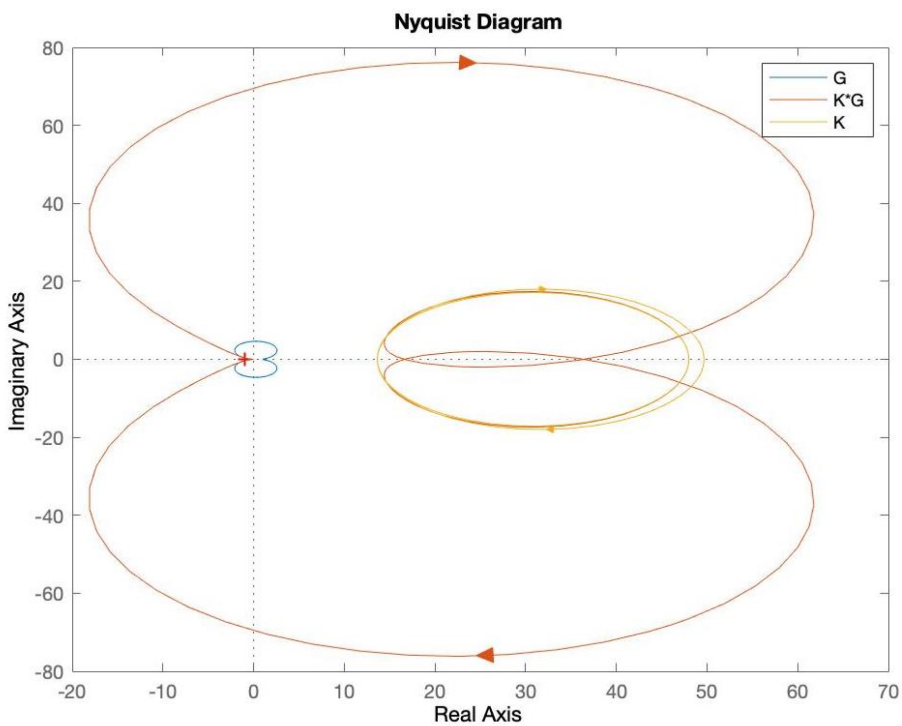


Figure 62: Nyquist plot of Lead-Lag Controller w/ Proportional Gain of 200

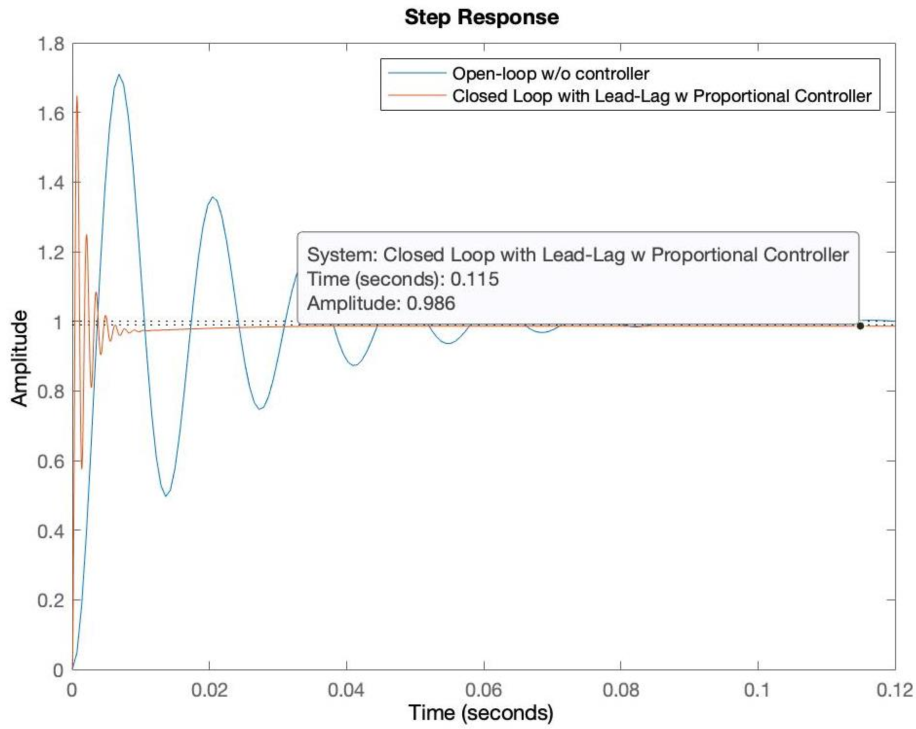


Figure 63: Step-response of Lead-Lag Controller w/ Proportional Gain of 400

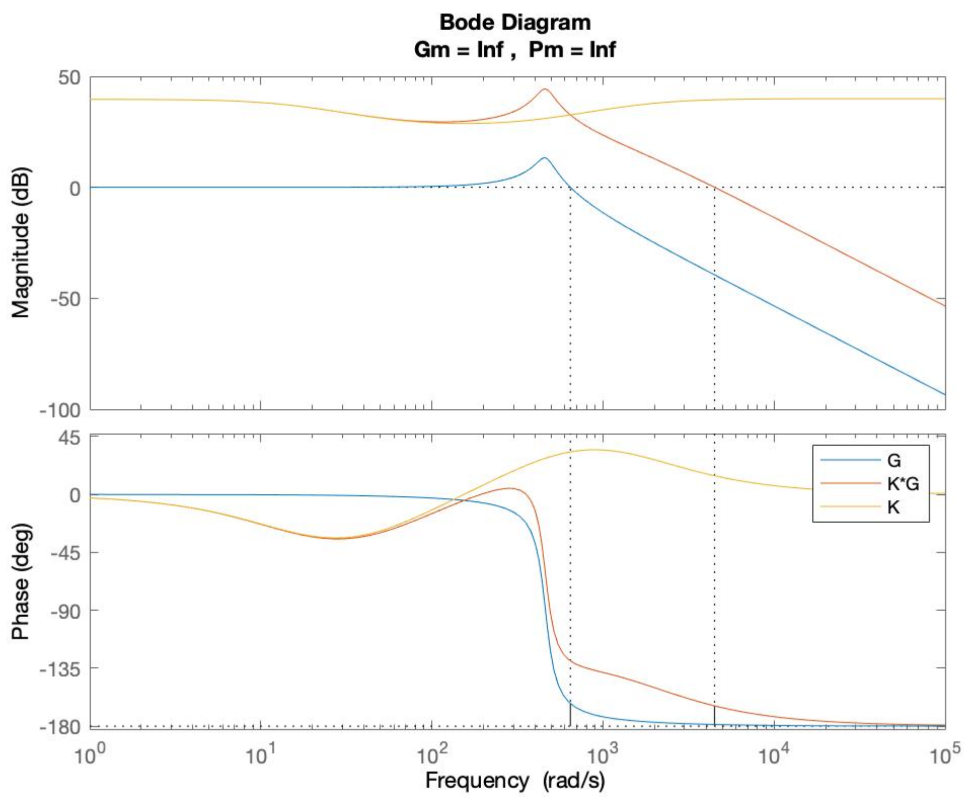


Figure 64: Bode plot of Lead-Lag Controller w/ Proportional Gain of 400

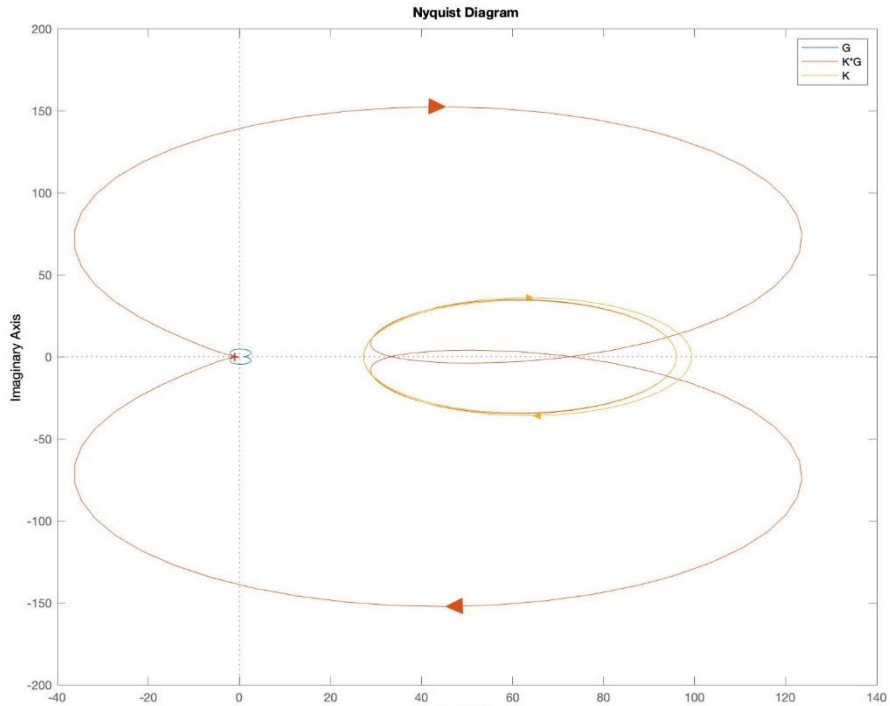


Figure 65: Nyquist plot of Lead-Lag Controller w/ Proportional Gain of 400

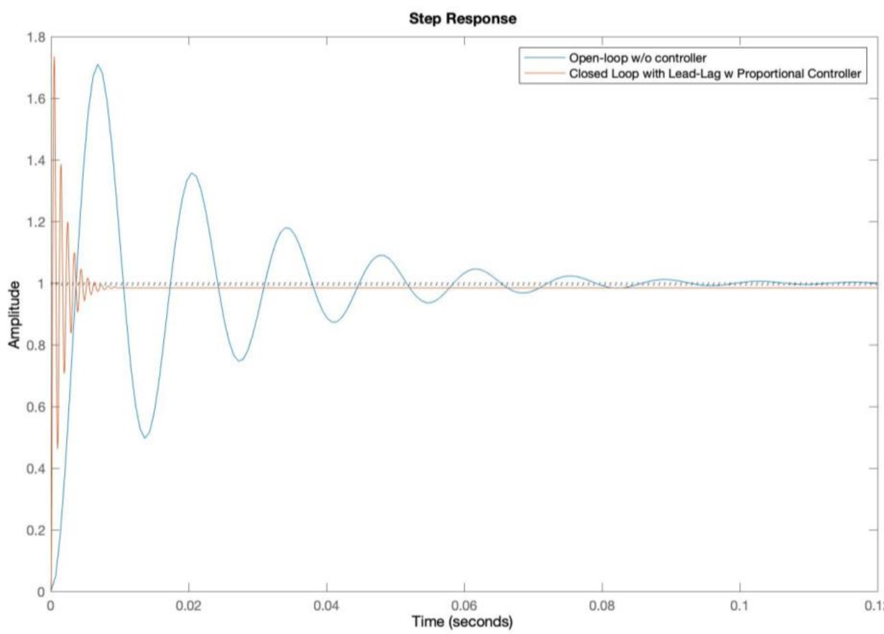


Figure 66: Step-response of Lead-Lag Controller w/ Proportional Gain of 800

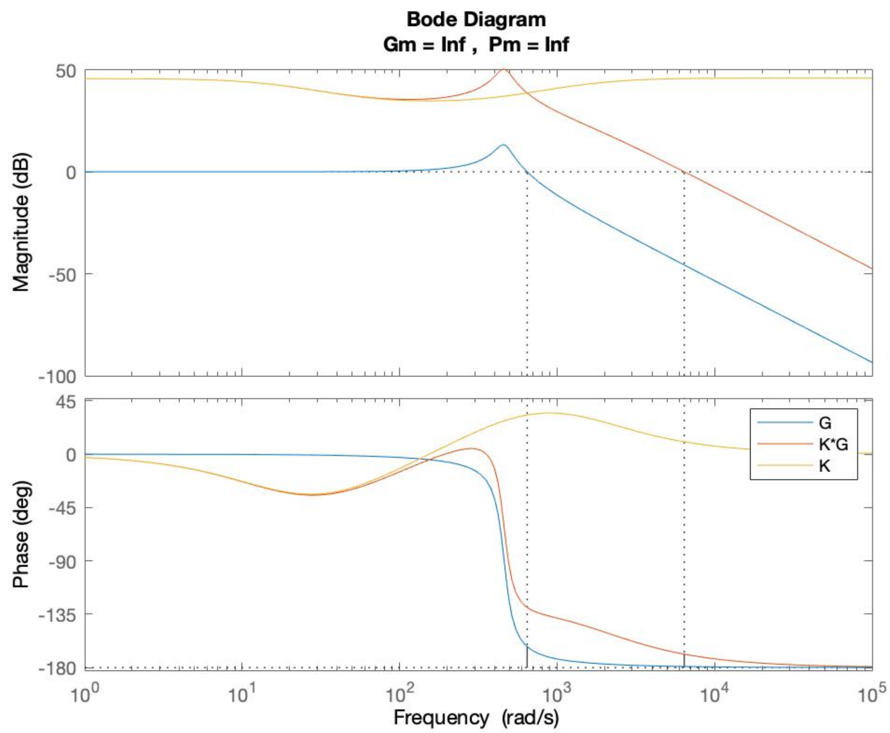


Figure 67: Bode plot of Lead-Lag Controller w/ Proportional Gain of 800

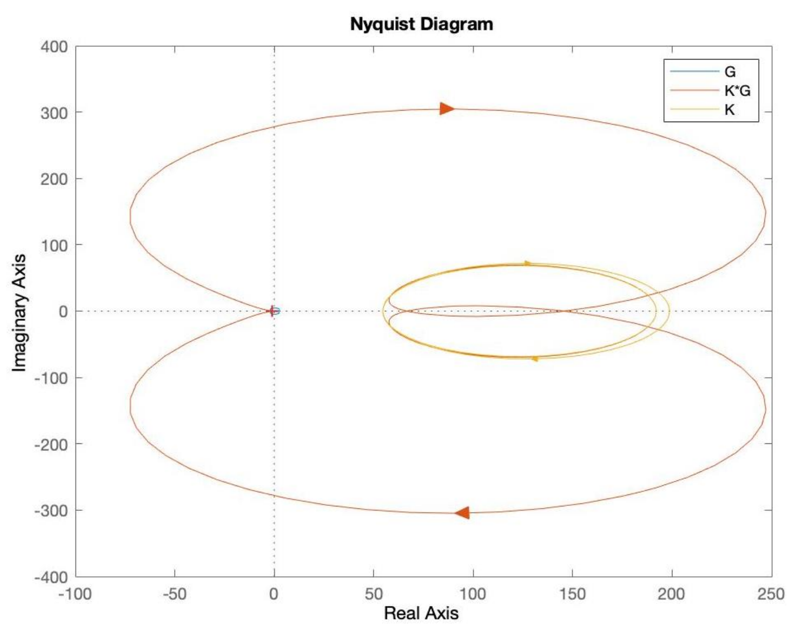


Figure 68: Nyquist plot of Lead-Lag Controller w/ Proportional Gain of 800

3.3.2.4 Lead-Lag Compensator Design Testing

MATLAB Simulation

<u>Case I:</u> R=100 Ω, L=100mH, C=47μF

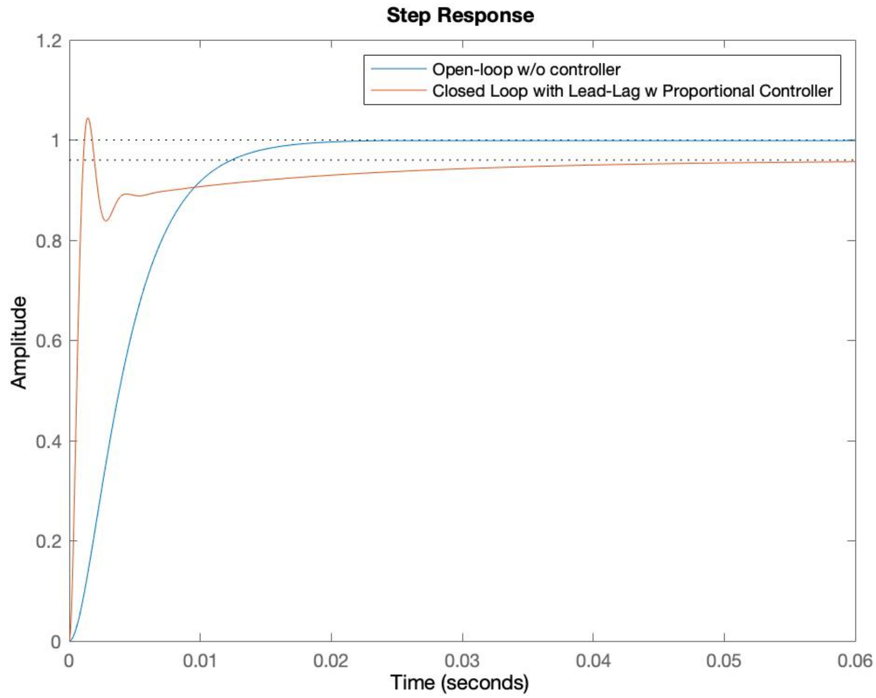


Figure 69: Step response of Lead-Lag Controller w/ 100 Ω

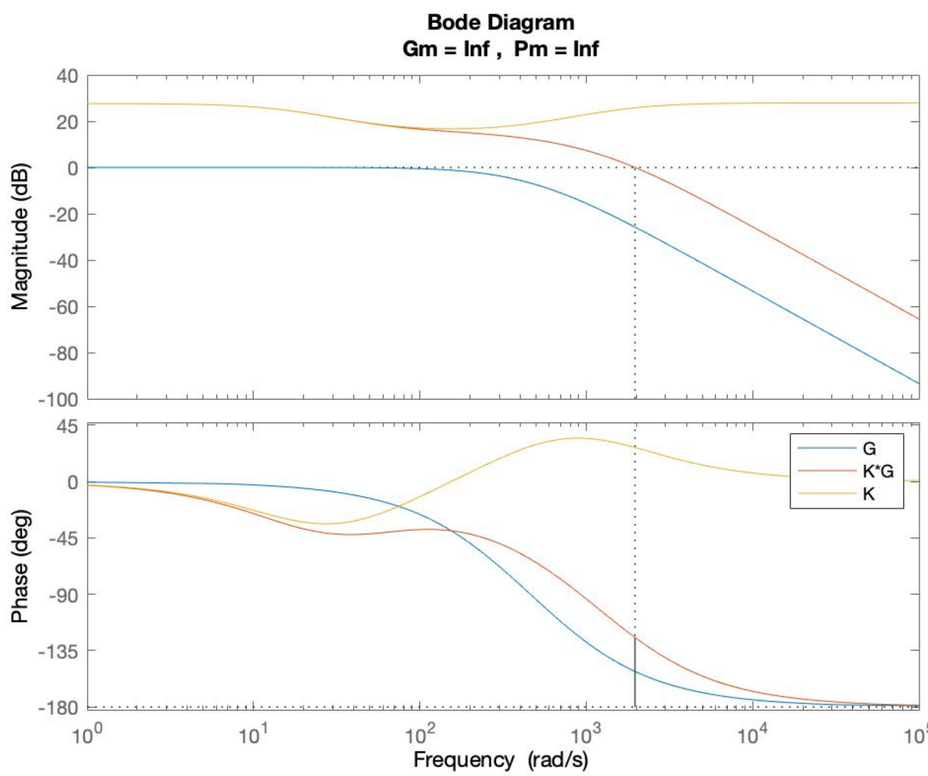


Figure 70: Bode plot of Lead-Lag Controller w/ 100Ω

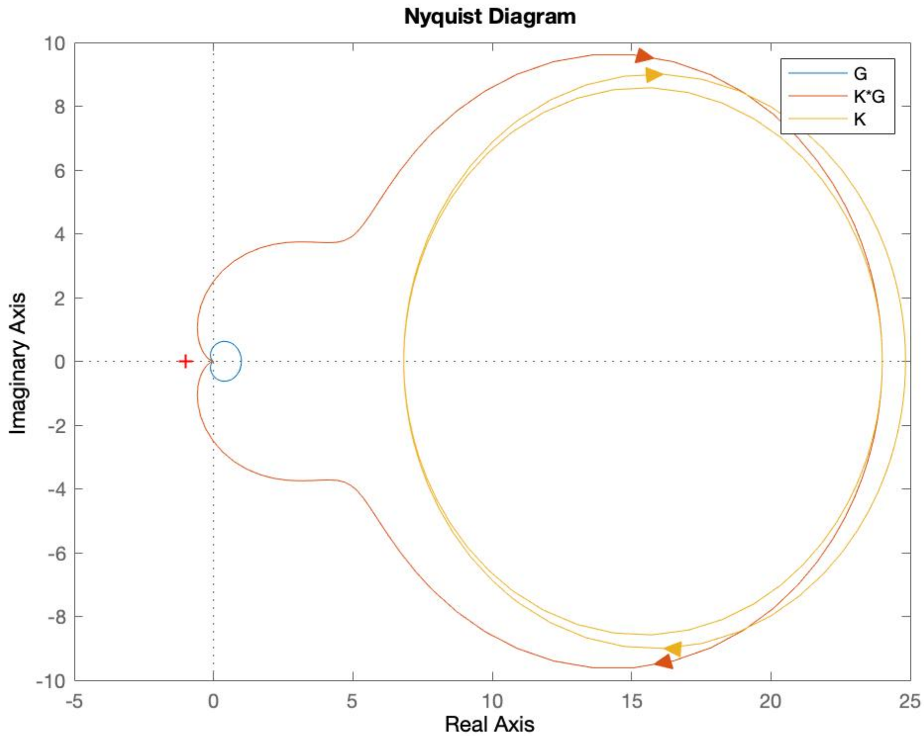


Figure 71: Nyquist plot of Lead-Lag Controller w/ 100Ω

<u>Case II:</u> R=10 Ω, L=10mH, C=47μF

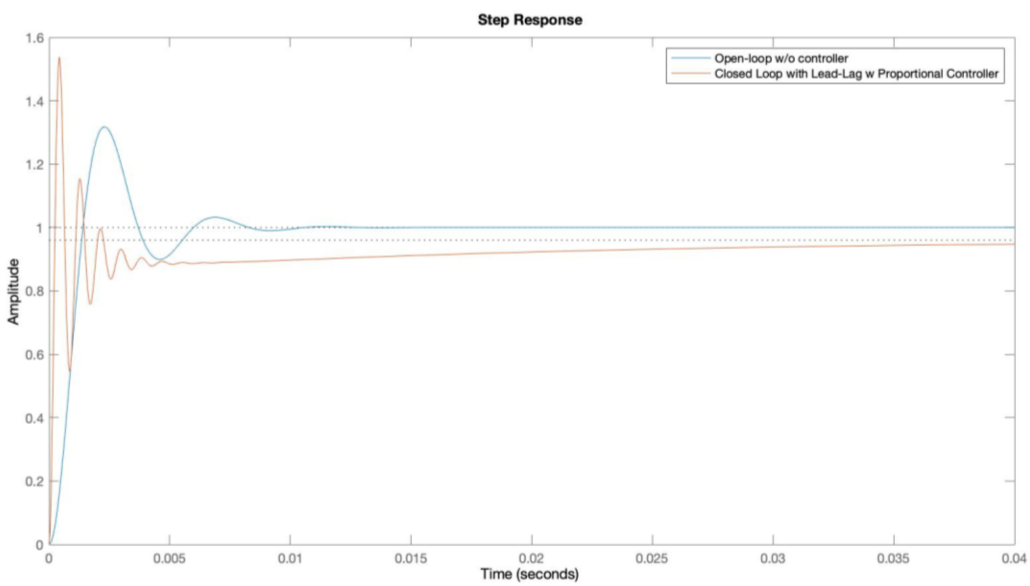


Figure 72: Step-response of Lead-Lag Controller w/ 10mL

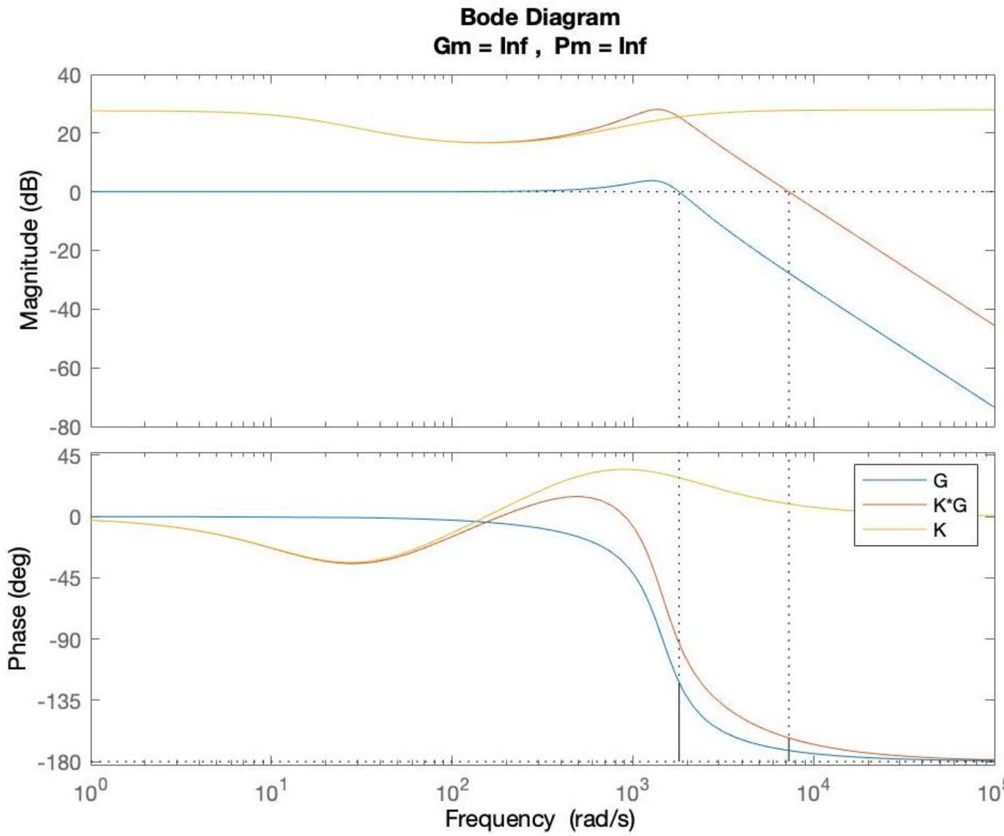


Figure 73: Bode plot of Lead-Lag Controller w/ 10mL

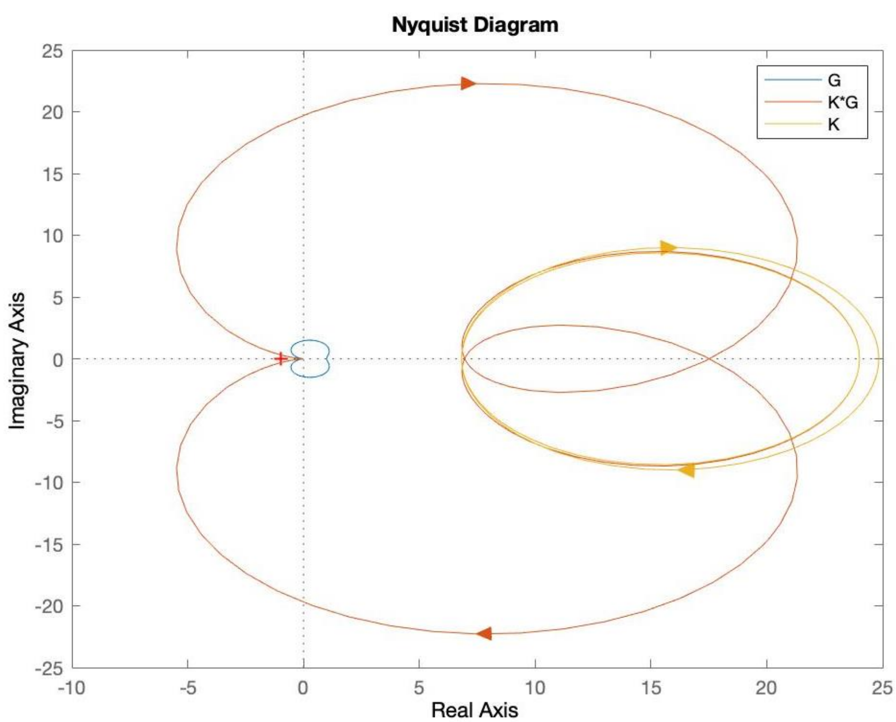


Figure 74: Nyquist plot of Lead-Lag Controller w/ 10mL

Case III: R=10 Ω, L=100mH, C=4.7μF

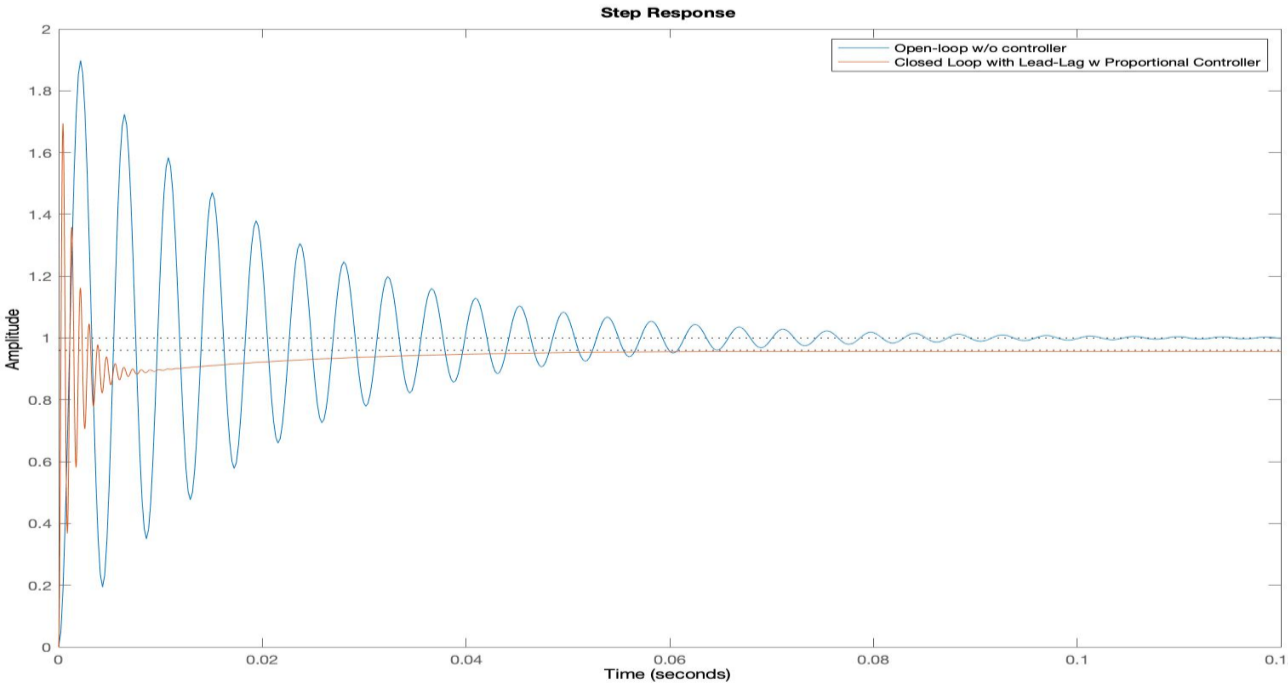


Figure 75: Step-response of Lead-Lag Controller w/ 4.7μF

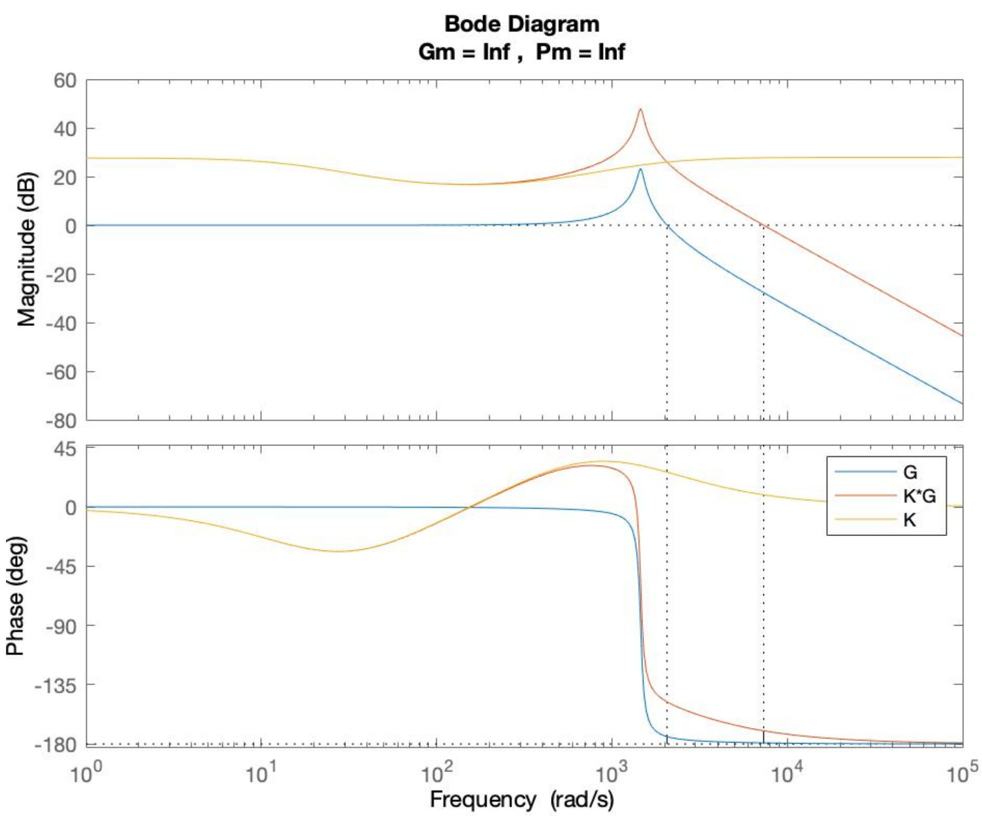


Figure 76: Bode plot of Lead-Lag Controller w/ 4.7μF

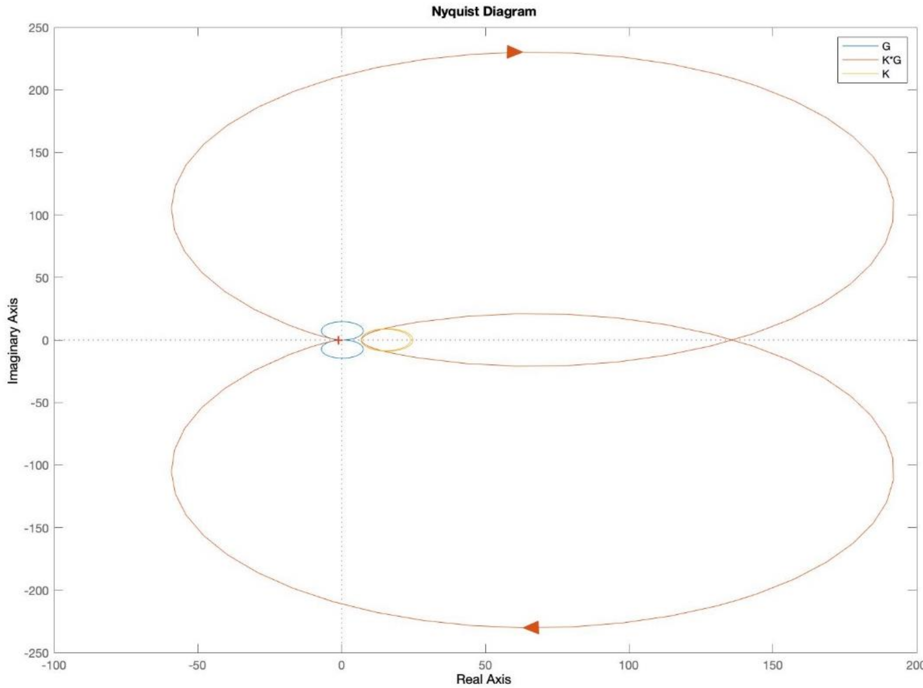


Figure 77: Nyquist plot of Lead-Lag Controller w/ 4.7µF

LTSpice Simulation

Case I: R=100Ω, L=100mH, C=47µF

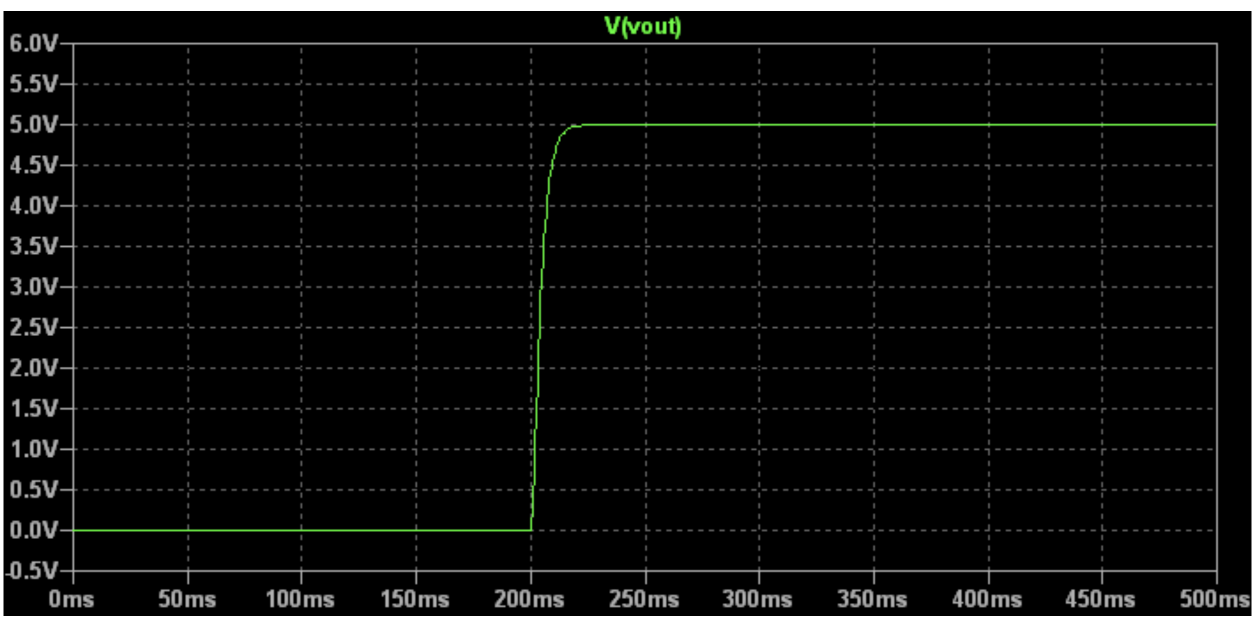


Figure 78: Step-response of LTSpice OL w/ 100 Ω

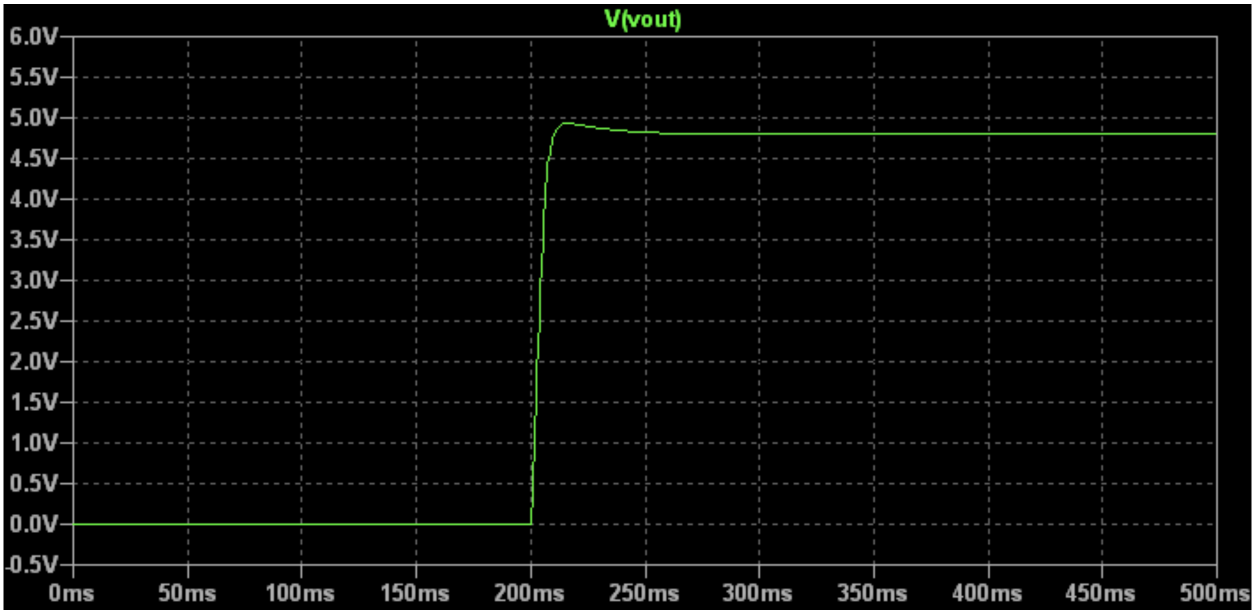


Figure 79: Step Response of LTSpice CL Lead-Lag Controller w/ 100Ω

<u>Case II:</u> R=10Ω, L=10mH, C=47μF

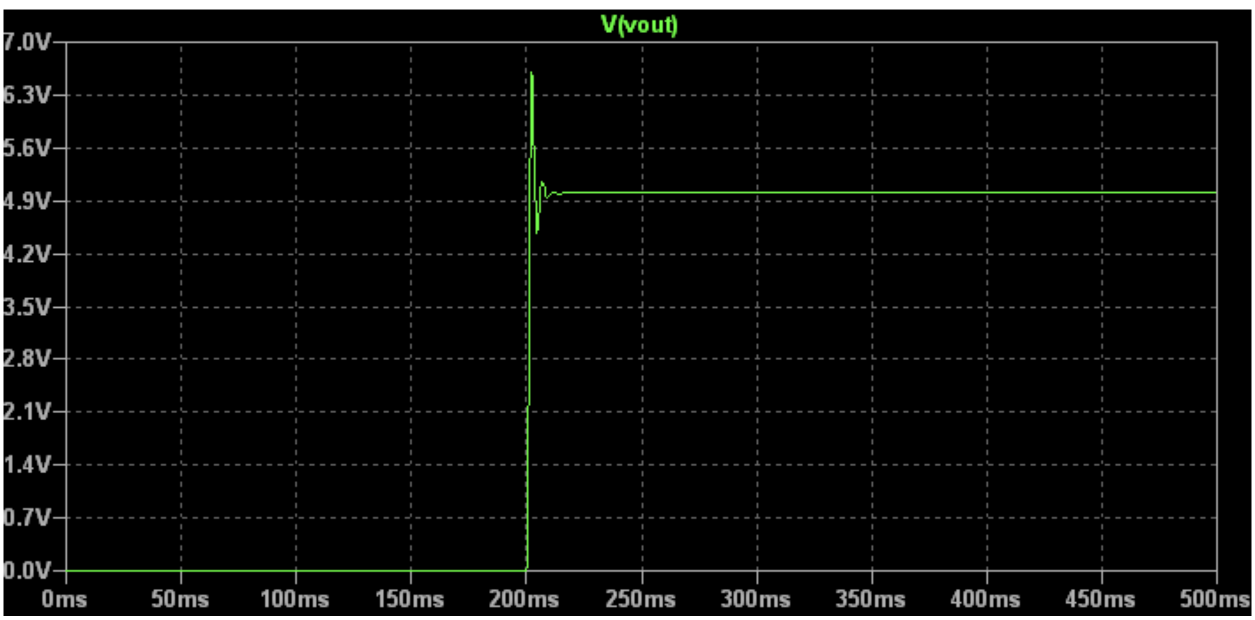


Figure 80: Step-response of LTSpice OL w/ 10mH

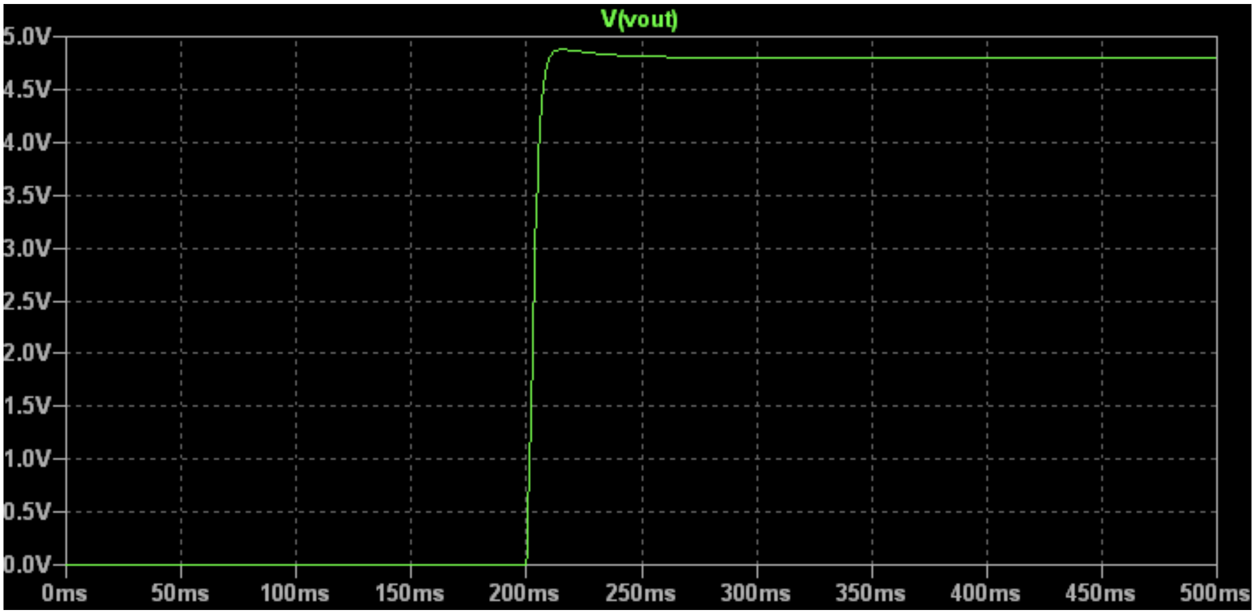


Figure 81: Step-response of LTSpice CL Lead-Lag Controller w/ 10mH

<u>Case III:</u> R=10Ω, L=100mH, C=4.7µF

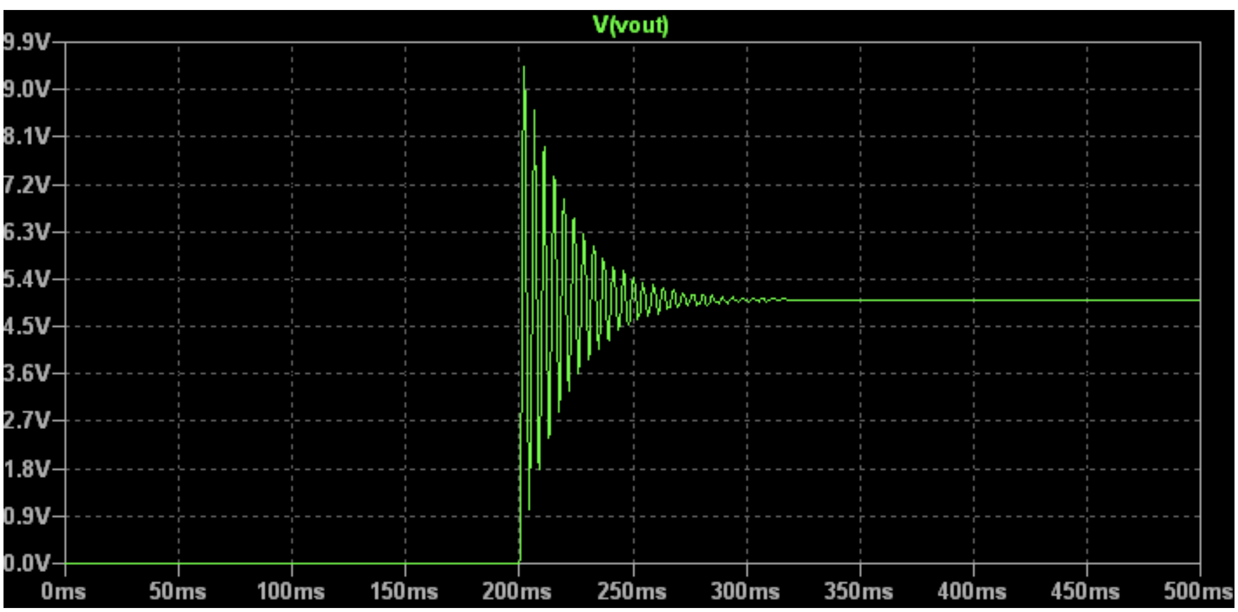


Figure 82: Step-response of LTSpice OL w/ 4.7µF

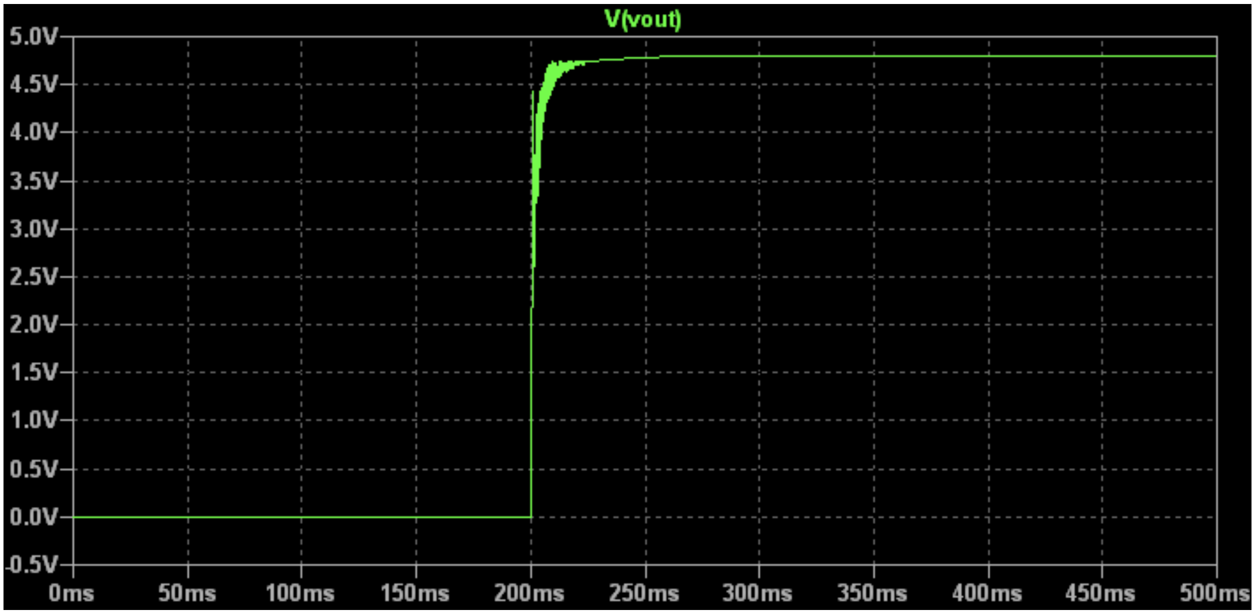


Figure 83: Step-response of LTSpice CL Lead-Lag Controller w/ 4.7µF

4. Discussion

4.1 LRC System Analysis

Knowing the impedance of the components of the circuit being: $Z_L = Ls$, $Z_R = R$, $Z_C = 1/Cs$, and using component values of L = 100mH, R = 10Ω, C = 47uF we are able to obtain the transfer function G(s) of the system to be:

$$G(s) = \frac{212765.96}{s^2 + 100s + 212765.96} \tag{24}$$

As we can see the simple circuit creates a second order system. From the obtained transfer function, we were able to simulate the system and generate the plots in the results section, and determine that the system is in fact stable.

Using the open loop transfer function obtained from the provided values we are able to observe that the step response has 70% overshoot and oscillates around 1 until it settles after around 0.1 seconds with no steady state error as seen in figure 14. It can be seen that the system is stable from the step response but to further confirm we look at the root locus plot. From simulation we were able to find the roots of the system to be at -50 ± 458.55i as it can be seen in figure 15. The placement of the poles being in the left hand plane shows us that the system is stable along with knowing the degree of the system is 2, we can observe that as K increases from 0 to infinity the roots only moving vertically along the imaginary axis and never cross over to the right hand plane; therefore the system won't become unstable unless externally acted upon.

To further investigate the stability of the system we can check the nyquist and bode plots, figures 16 and 17. From the nyquist plot we can see that the open loop system does not encircle the point (-1,0) which means that the system is stable according to the Cauchy theorem; however, from the bode plot it is evident that the system has a phase margin of only 17.6°. The phase margin implies that a time delay of 17° could be added to the system before system becomes unstable. Meanwhile, the gain margin being infinite in our system shows that if only gain is, no amount of gain increase will force the system to become unstable.

Comparing the MATLAB plots to the LTSPice Simulations in figures 18 and 19 offers confirmation that our model is correct as the step response can be seen to reach peak value of 1.71, and settles within 100ms. The phase margin from both simulations occur around 645rad/s at 17degrees.

After verifying the stability of the system, the effects of changing each parameter individually is explored by increasing each value by 5 times the original number. One again plots were generated for the sake of comparison.

In figure 20, it seems that increasing the inductor to 500mH causes the system to become more underdamped, causing oscillations continue for 5 times longer before resting at the steady state value. Increasing the capacitance to 235uF only slightly decreased the overshoot and increased the rise time, while maintaining a consistent settling time with the original. By far the best response came from increasing the resistor to 50**Ω**, with the smallest overshoot of only 13% and settling time of 0.025seconds. From an electrical perspective these results are consistent with expectations - as increasing the resistance decreases the current which will create a smaller inrush current on the inductor providing a more stable response; whereas, increasing the inductor without modifying the circuit to reduce the current will cause higher inrush current resulting in large voltage spikes. Finally, the increase in capacitance will cause higher current spikes when the circuit is initially connected which causes the rise time to spike.

Comparing the step response with figures 21 and 22, the root locus and nyquist plots respectively, it is apparent that the system with 5 times increase in resistance is indeed the most stable as the roots are further in the left hand plane, and the increased inductance moves the roots closer to the origin. The increased capacitance can be seen to occur closer to the real axis compared to the original which can be is an indication the system becoming closer to being critically damped.

Observing figure 23 and table 1 for the phase margins between the systems we are again able to verify that the higher resistance increases the stability of the system by roughly 6 times, while the higher inductor dropped the stability by over half.

4.2 Negative Feedback

Recalling, negative feedback is a process in which the output of the system is fedback to system input as an error signal. The feedback process in an electronic circuit is achieved as explained in the theory section. The objective of this section was to check if a closed-loop system without a controller has an improved step-response as compared to an open-loop system.

Figure 24 in Results shows the step-response for the open-loop and closed-loop system. The closed-loop system response is without a controller. Comparing the two step-responses, it is evident that the closed-loop system has a greater steady-state error, since the final value of the closed-loop system is half of that achieved by the open-loop system. Also, the closed-loop system has no overshoot meaning the system is relatively more stable. Figure 25 in Results show the Root locus plot for both the open and closed-loop system. Analyzing the plots, both systems have the same settling time since the real part of the poles of each system have the same value.

All in all, analyzing the step-response and the root locus of the open-loop and closed-loop system, we can conclude that the closed-loop system without a controller does not significantly improve the system's behaviour. This may be further confirmed through the MATLAB simulation for step-response as shown in figure 24 - without a controller, the closed-loop system is relatively stable as there is no overshoot, but the steady-state error increased significantly.

4.3 PID Control

A PID controller is made of three components; a proportional, integral and derivative controller. When combining these three we are able to control all attributes of a response signal in terms of overshoot, stability, steady-state, and response time.

From theory section 2.3.1, it is possible to combine the three op-amp circuits to create a PID controller with full control of the gains used. However, the controllers may be run separately to determine what effects on the system. Starting with a simple P controller, then a PD controller, and finally a PI controller.

First, to create the P controller K_i and K_d are set to zero, and K_P is independently varied from 2 to 20. The resulting figure 26, it is apparent that for a small K_P value the oscillations are slightly greater than that of the open loop system, and the steady-state error is huge. As K_P is increased the steady-state error decreases till it reaches zero; however, the overshoot and oscillations of the system increase dramatically.

Next, a PD controller is devised by setting K_P to 4, K_i to 0 and then varying K_d from 2 to 20. Figure 27 shows that regardless of the value of K_d the steady state error is constant, and always below the desired value. It is evident that the K_d value is directly proportional to the settling time.

Finally, a PI controller is done by setting K_P again to 4, K_d to 0 and then varying K_i from 2 to 20. As seen in figure 28 a larger K_i gain causes the response to reach zero steady-state error faster than a small K_i.

From the independent tests we can conclude that the proportional gain has the greatest effect of driving the response to the input signal - as in the larger the K_P, the faster the system will rise. In contrast, the derivative gain acts against K_P by pushing the signal down to balance the over shooting that occurs; however, a larger K_d to cancel the oscillations will also result in a longer settle time. Finally, the integral gain is used to minimize steady-state error, while also decreasing settling time.

To obtain the desired output as seen above in figure 29 the gain values selected were; $K_P = 4$, $K_i = 14$, $K_d = 12$. From these desired gain values, we then determined real world component values to construct the circuitry in LTspice which can be found in the appendix figure 84. By using true component values the gain values varied slightly from the ideal case which came out to $K_P = 4.03$, $K_i = 14.18$, $K_d = 11.985$.

4.4 Lead-Lag Compensator

The objective of this section was to design a Lead-lag compensator for the closed-loop system to improve the system's response behaviour. Before designing the lead-lag compensator for a simple RLC circuit, the effects of Lead Compensator and Lag Compensator on our closed-loop system was analyzed by plotting bode and nyquist plots for the open-loop systems with the respective controller. The following text will discuss the conclusions drawn based on the figures presented in the Results section.

Lag Compensation

Initially, we analyzed a Lag Compensator. Lag compensator essentially yields an improvement in the steady-state accuracy at the expense of increasing the transient response time.

The effect of lag compensator at different frequency ranges was investigated using MATLAB and LTSPice as mentioned in the Theory section of Lead-Lag Compensation. The results of this experiment are included in the Result section under Lag Compensator design, labelled as Case I and Case II. The difference between the two cases is the distance between the pole and the zero introduced by the lag compensator. The corner frequencies are further apart in Case II as compared to Case I.

Figure 31 shows the step response of the system with the addition of the Case I-Lag compensator to the system. The open-loop system without the lag compensator had a phase margin of 17.6 degrees. The lag compensator was added to the original system with corner frequencies of 1000 and 2000 [rad/s]. The bode plot in figure 32 shows the effects of an added lag compensator on the original open-loop system. The compensator's gain is unity at low frequencies which means the lag compensator didn't improve the steady-state accuracy of the original closed-loop system. Moreover, the gain of the lag compensator at high frequencies is not 1, meaning compensator has affected the transient response and the stability characteristics of the system.

The effect on the transient response is evident in the MATLAB simulation and LTSPice, as shown by figures 32 and 37. A Lag compensator shifts the root locus of the original closed-loop system to the right, slowing down the transient response of the system by increasing settling time. The relative stability of the system is reduced by the addition of the lag compensator; the nyquist and bode plots of the system show that the phase margin of the new open-loop system has reduced to 2.95 degrees from 17.6 degrees. The phase margin is a measure of how much delay a system can handle before it becomes unstable.

Figure 34 shows the step response of the system with the addition of the Case II-Lag compensator to the system. The lag compensator in this case was added to the original system with corner frequencies of 1000 and 3000 [rad/s]. The bode plot in figure 35 shows the effect of the added Case-II lag compensator on the original open-loop system. The negative phase introduced by this lag compensator in the defined frequency range has reduced the phase margin of the system from 17.6 degrees to -2.69 degrees. The negative phase margin is an indication that the lag compensator has shifted the root locus of the original system to the unstable region - the left side of the complex s-plane. This explains the unstable behaviour shown by the MATLAB simulation and LTSpice, in figures 34 and 38 respectively.

Lead Compensation

Next, the lead compensator is explored which essentially yields an improvement in the transient response of the system. The effects of a lead compensator at corner frequencies of 1000 and 2000 [rad/s] was investigated using MATLAB and LTSpice as mentioned in the Theory section 2.3.2., and results of this experiment are included in the Result section under Lead Compensator design.

Figure 39 shows the step response of the system with the addition of the lead compensator to the system. The open-loop system without the lag compensator had a phase margin of 17.6 degrees, and the lead compensator was added to the original system with corner frequencies of 1000 and 2000 [rad/s]. The bode plot in figure 40 shows the effect of the added lead compensator on the original open-loop system. Gain is unity at low frequencies meaning the lead compensator does not affect the steady-state accuracy of the original closed-loop system. Moreover, at high frequencies the gain is no longer 1 meaning the lead compensator has affected the transient response and the stability characteristics.

The effect on the transient response can be noticed in the MATLAB simulation as shown by figure 39. A Lead compensator increases the crossover frequency which decreases the settling and rise time of the system's response, thereby resulting in a faster transient response. The relative stability of the system is also increased by the addition of the lead compensator, as the nyquist and the bode plot of the system show that the phase margin of the new open-loop system has increased to 42.1 degrees from 17.6 degrees. The response of the system when the PID controller was improved in all conditions. Overshoot reduced from 70% down to roughly 20%, settling time also decreased by 50ms.

Lead-Lag Compensator Design

Moving on from the analysis of the Lag and Lead compensators, s Lead-Lag compensator for our simple LRC circuit was designed to improve its transient response and reduce the steady-state error independently. During the design process, the transient response of the system was improved first, followed by improvement of the steady-state accuracy. The disadvantage in improving the steady-state error first is that the improvement of the transient response in some cases yields some decay in the improved steady-state error.

Note: Calculations for the design process can be found in Appendix, for your reference.

To improve the transient response of the system, a lead compensator is added to the original system. Recalling from the theory section, a lead compensator adds a positive phase at the specified corner frequencies and adds a gain at high frequencies. The positive phase, if added at the gain crossover frequency of the original open-loop system, will increase the phase margin of the new system. The increase in phase margin makes the new system relatively more stable. Also, the added gain at high frequencies increases the crossover frequency, which helps lower the settling and rise time, resulting in a faster transient response by the system. Keeping these facts in mind, the lead compensator was added at the phase margin frequency of the original open-loop system to not only improve the transient response but also make the system relatively more stable. The phase margin of the original open-loop system is 17.6 degrees and the gain crossover frequency 645 [rads/s].

Figure 42 shows the original and improved transient response of the system. Figure 43 is the bode plot of the original open-loop system and the new open-loop system. It is apparent that the phase margin of the new open-loop system has increased from 17.6 degrees to 75.7 degrees, resulting in greater relative stability of the closed-loop system. As expected, the transient response of the system with the lead compensator is much faster than the original closed-loop system with no controller as shown in figure x.

Next, to improve the steady-state accuracy of the system, a lag compensator is added to the improved system. Recalling from the theory section, a lag compensator adds a negative phase at the specified corner frequencies and adds a gain at low frequencies. The negative phase, if added at the improved gain crossover frequency of the improved open-loop system, decreases the phase margin of the improved system, and consequently yields some decay in the improvement of transient response, which was designed first. Therefore, care had to be taken to ensure that the lag compensator is applied at a frequency lower than the improved gain crossover frequency so that the transient response and stability characteristics of the system are not significantly altered. Keeping these facts in mind, the lag compensator was added at a frequency of 61.3 [rads/s] which is well below the improved gain crossover frequency of 524 [rads/s].

Figure 46 is the bode plot of the open-loop system and the improved open-loop system. It is evident from the magnitude plot that the lag compensator has increased the magnitude of the open-loop system at low frequencies, resulting in an reduced steady-state error. The addition of the lag compensator to the system also improved the phase margin from 75.7 degrees to infinity. This means the improved system is relatively more stable and can withstand any amount of time-delay. Figure 45 is the step-response of the improved system with the Lead-Lag controller. Comparing this step-response to that in figure 42, it seems the addition of the lag compensator yields a slight decay in the previously improved transient response.

Finally, a proportional gain controller is added to the system to push the system's response to 1 by testing various K_p. The proportional gain Kp has certain limitations depending on the gain margin of the improved system. Recalling from theory section, gain margin is a measure of how much additional gain can be added to the system before the system becomes relatively unstable. Therefore, increasing Kp decreases the phase margin of the system as the nyquist plot of the improved open-loop system is stretched out. Therefore, the objective of utilizing proportional gain is to get as close to the desired value as possible, while ensuring that the phase margin of open-loop system with the lead-lag compensator is improved as compared to the original open-loop system without lead-lag compensator.

Figures 48 to 68 show step-responses, bode plots and nyquist plots for different proportional gains ranging from 10-800. It is evident that proportional gains within the range of [10,200] have an improved phase margin as compared to the original open-loop system without a lead-lag compensator. The step response achieved within this proportional gain range is not perfect but the system is relatively more stable. Proportional gains in the range of (200, 800], have a better step-response but the systems are relatively less stable as compared to the original open-loop system. Our lead-lag compensator has a proportional gain of 100 - it provides a step response of 0.96 in 0.06 seconds and has a phase margin of 28.9 degrees. The original open-loop system with no compensator provided a step response of 1 in 0.1 seconds and had a phase margin of 17.6 degrees. Therefore, it is fair to say that addition of the lead-lag compensator has improved transient response and made the system relatively more stable.

Lead-Lag Compensator Design Testing

Following the design of our lead-lag compensator, it was tested for robustness with different plants. The step-response, bode plot and nyquist plot for each plant were plotted in MATLAB and LTSpice simulations were conducted. These can be found in the Lead-Lag Compensator Design Testing section under Results for your reference.

Robustness is essentially a measure of system stability. Gain and Phase margin are the two quantities used to indicate the margin the system has before it becomes unstable. The three plants are referenced as Case I, Case II, and Case III. In each plant, two of the components of the LRC circuit are the same as the original plant while the third component is increased by a factor of 10.

In Case I, the resistance was increased by a factor of 10, from 10Ω to 100Ω. Figure 69, 70 & 71 are the step-responses of the open-loop and closed-loop system, bode and nyquist plots for the open-loop system with and without the designed lead-lag compensator. The step-response of this plant shows that the closed-loop response of the system was much faster than the open-loop system without the controller, though not the best step-response. However, bode plot shows that the open-loop system with the designed lead-lag compensator had a better phase margin than the open-loop without the compensator, making the closed-loop system relatively more stable. Also, the nyquist plot shows that the open-loop system without the designed lead-lag compensator had a larger gain margin than the open-loop system with the designed lead-lag compensator, which means that the system without the compensator can withstand more gain before becoming unstable as compared to the system with the designed lead-lag compensator.

In Case II, the inductance was increased by a factor of 10, from 100mH to 10mH. Figure 72, 73 & 74 are the step-response of the open-loop and closed-loop system, bode and nyquist plot for the open-loop system with and without the designed lead-lag compensator. The step-response of this plant shows that the closed-loop response of the system was much faster and had more overshoot than the open-loop system without the controller. The bode plot shows that the open-loop system without the compensator had a better phase margin than the open-loop with the designed compensator, making the closed-loop system relatively less stable. The nyquist plot shows that the open-loop system without the designed lead-lag compensator had a larger gain margin than the open-loop system with the designed lead-lag compensator, which means that the system without the compensator can withstand more gain before becoming unstable as compared to the system with the designed lead-lag compensator. All in all, the compensator in this case made the system less robust as phase margin and gain margin of the open-loop system without the compensator were relatively better. Therefore, the addition of the compensator made the system relatively less stable.

In Case III, the capacitance was increased by a factor of 10, from 47μF to 4.7μF. Figure 75, 76 & 77 are the step-response of the open-loop and closed-loop system, bode and nyquist plot for the open-loop system with and without the designed lead-lag compensator. The step-response of this plant shows that the closed-loop response of the system was much faster and had less overshoot than the open-loop system without the compensator. The bode plot shows that the open-loop system with the designed compensator had a better phase margin than the open-loop without the compensator, making the closed-loop system relatively more stable. The nyquist plot shows that the open-loop system with the designed lead-lag compensator had a larger gain margin than the open-loop system without the compensator, which means that the system with the designed compensator can withstand more gain before becoming unstable as compared to the system without the compensator. All in all, the compensator in this case made the system more robust as phase margin and gain margin of the open-loop system with the designed compensator were relatively better. Therefore, the addition of the compensator made the system relatively more stable.